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MATHEMATICAL MODEL BETWEEN BASELINE AND MINIMUM VISA TIME WITH HIGH-PRECISION GNSS TECHNOLOGY

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Abstract

In relation to the observation of baselines with high precision GNSS receivers, there are few studies where the minimum observation time of a baseline is established with respect to its length, in certain guides of recognized GNSS receiver brands certain observation times are recommended depending on the baseline length, But it is not supported why, nor how such a recommendation is reached.

The Peruvian technical standard only establishes a minimum observation time regardless of the length of the baseline.

In terms of productivity and efficiency, establishing a relationship between the minimum observation time for a given baseline length is reasonable.

In the present study, the relationship between the minimum visa time (observation) required for a given baseline length is found, through a mathematical relationship.

For this purpose, different baselines were observed at different time intervals, then these sets of measurements were subjected to statistical methods to eliminate observation errors.

With these results the minimum observation time is related to its baseline length, obtaining a linear relationship, these results are similar to the recommendations in the geodetic receiver guides.

Therefore, the results in the present study are satisfactory and coherent and also suggestive to the realization of further studies using other brands of receptors, suggesting observations at larger and different lengths of baselines.

Keywords: Mathematical Models, GNSS Technology, High Precision.

1. Introduction

The importance in the observation of geodetic points of order "C" lies in their level of accuracy (maximum of 10 mm); thus becoming a widely used tool for the control and development of various engineering projects.

The Geodetic Technical Standard, in Chapter 3, establishes certain considerations for the collection of data from geodetic points of order "C":

For the collection of data from all geodetic points of order "C", the static relative method will be used, these will be obtained with the support of at least one geodetic point, either of order "0", order "A" or order "B" at the national level, which are equidistantly separated, at a distance not exceeding 100 km to the geodetic point to be established, considering the continuous observation time not less than 900 records or epochs (of coincidence with the base), not less than one (1) second nor more than five (5) seconds of synchronization (with the base), with an elevation of the mask not greater than fifteen (15) degrees above the horizon and with the permanent tracking of not less than 4 satellites.

The Geodetic Technical Standard establishes a minimum observation time without considering the length of the baseline.

1.1 Description of the problem

In geodesic networks located in the sierra, or high jungle of our territory, in which they present a steep topography and narrow valleys, the geodesic points are usually located on mountain tops; producing that the time of transport, installation and observation of the receivers demand a large amount of time, limiting themselves to reading few baselines in the course of the day.

In terms of productivity and accuracy, it is preferable to make observations at shorter time intervals than those indicated in the geodetic technical standard as long as the required precision is maintained.

2. Objectives

2.1 General objective

Determine a mathematical model between baseline and minimum visa time with High Precision GNSS technology.

2.2 Specific objectives

 Obtain a set of observations made at different time intervals at different baseline lengths with high-precision GNSS technology.

 Obtain a set of valid observations by debugging the erroneous observations made by high-precision GNSS technology through mathematical procedures.

3. Mathematical model

3.1 Initialanalysis of baseline observations (measurements)

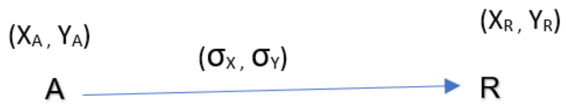
 Baseline measurement scheme: planimetric coordinates (East, North) are used for the present study.

Table 1. Coordinates for a baseline and its respective standard deviation

GPS	Point	This	North	p(E)	s(N)
Base	A	X_A	AND$_{TO}$	0	0
Rover	R	X_R	The$_R$	S_x	S_y

Source: (Mendoza Dueñas, 2020)

Figure 1. Basic model of a baseline



$$(L_1, L_2) = (\Delta X, \Delta Y) = (X_R - X_A, Y_R - Y_A)$$

Source: Authors.

• L1 and L2 are the results of satellite observations, product of the relative method.

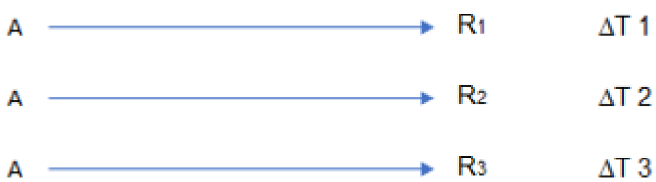
• σx2; σ and 2 are the variances of observations L 1 and L2 respectively.

• Model with a reading at a given ΔT1.

• For different ΔT, the coordinates differ as seen in the processing results in the first stage.

• The same baseline is read at different time intervals, with fixed coordinates in A, obtaining different reading coordinates: R1, R 2 and R3

Figure 2. Baseline reading at different time intervals

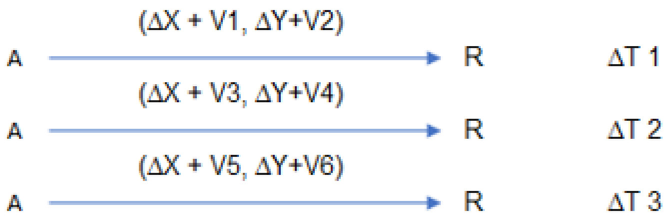


Source: Authors.

3.2 Waste

• To obtain unique coordinates, the so-called "Vi" residues are added:

Figure 3. Waste vi

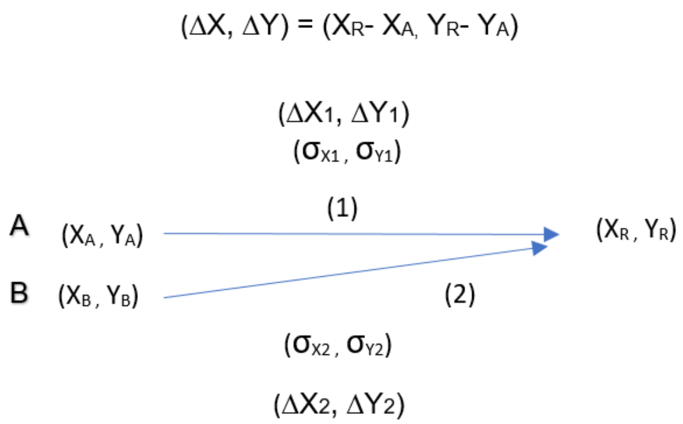


Source: Authors.

• The residues Vi would be related to the standard deviation of the measurement.

• We will generalize the model for different baselines with a common point of arrival.

Figure 1. Generalization of the mathematical model



Source: Authors.

3.3 System of linear equations approach

• Figure N°26 presents the following equations:

Baseline (1)

$$R_x = X_A + (\Delta X_1 + V_1) \rightarrow V_1 = R_x - (X_A + \Delta X_1)$$

$$R_y = Y_A + (\Delta y_1 + V_2) \rightarrow V_2 = R_y - (Y_A + \Delta Y_1)$$

Baseline (2)

$$R_x = X_B + (\Delta X_2 + V_3) \rightarrow V_3 = R_x - (X_B + \Delta X_2)$$

$$R_y = Y_B + (\Delta Y_2 + V_4) \rightarrow V_4 = R_y - (Y_B + \Delta Y_2)$$

• Unknowns:

R = (Rx, Ry)

• Equations of Condition:

V1	=	(1) R_x	+	(0) Association	+	(- X_A - ΔX_1)
V2	=	(0) R_X	+	(1) Association	+	(- Y_A - ΔY_1)
V3	=	(1) R_x	+	(0) Association	+	(- X_B - ΔX_2)
V4	=	(0) R_X	+	(1) Association	+	(- Y_B - ΔY_2)

3.4 Defining matrices

$$X = \begin{bmatrix} R_x \\ R_y \end{bmatrix} \; V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \; U = \begin{bmatrix} -X_A & -\Delta x_1 \\ -Y_A & -\Delta y_1 \\ -X_B & -\Delta x_2 \\ -Y_B & -\Delta y_2 \end{bmatrix}$$

[A] $_{4x2}$ = Matrix of coefficients A

[X] $_{2x1}$ = Unknown vector X

[U] $_{4x1}$ = Vector of independent terms U

[V] $_{4x1}$ = Residue vector V

3.5 Inconsistent linear systems

The adjustment of baselines, requires the approach and solution of linear systems, whose characteristics are:

a) The number of equations exceeds the number of unknowns (systems over determined)

b) The vector of independent terms does not belong to the column space of the coefficient matrix

c) They are inconsistent because of observational errors, which affect the vector of independent terms, preventing it from belonging to the column space of the coefficient matrix.

The inconsistency of the linear system makes it impossible to find a solution by classical methods (Gaussian elimination, for example).

Baseline observation equations are proposed, the solution to the inconsistent linear system arises by the application of Weighted Least Squares.

Considering a system of n linear equations with m unknowns: AX = U

$[A]_{nxm}$ = Matrix of coefficients A
$[X]_{mx1}$ = Unknown vector X
$[U]_{nx1}$ = Vector of independent terms U

AX = U

$$\begin{bmatrix} a_{11} & a_{12} & \dots & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & \dots & a_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & \dots & a_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}$$

From the dimension theorem: **d = m – r**
d = rank defect
m = number of unknowns
r = R(A), range of matrix A
Yes:
R(A) = m; supports single solution (full-range matrix A)
R(A) < m; the system has infinite solutions (poor rank A matrix)
AX = U, Has solution if and only if the vector U can be expressed as a linear combination of columns A(j), with scalars x j, for j = 1, m. If such a linear combination does not exist (because scalars xj do not exist), the vector of independent terms U does not belong to the column space of matrix A (ECA).

The linear system is then said to be inconsistent; that does not support solution. However, it is possible to find a solution by adding to the vector U a vector V (vector correction or vector error) such that: AX = U + V.

$$\begin{bmatrix} a_{11} & a_{12} & \dots & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

If we choose V such that AX is the orthogonal projection of U onto the ECA, V is a vector of minimum length (known as Gauss's least squares principle):

$$V^T V = \sum_{i=1}^{n} v_i^2 = mínimo$$

When and only when V is perpendicular to ACE.

Let X ≠ 0 be the least squares solution vector of the linear system: AX = U + V

If AX is perpendicular to V, we have that the scalar product is zero:

$$(AX)^T V = 0$$

But V = AX – U; Then

$$\mathrm{X^T A^T (AX - U) = 0}$$
$$\mathrm{X^T \left(A^T AX - A^T U\right) = 0}$$

Since X ≠ 0, it must necessarily be fulfilled:

$$A^T AX - A^T U = 0 \qquad \rightarrow \qquad A^T AX = A^T U$$

I mean:

A TAX = A T U is the system of normal equations, where the normal matrix N = A T A is symmetric of order mxm (N T = N).

The least squares solution is:

$$X = N^{-1} A^T U$$

And it will be unique if and only if the determinant of the normal matrix is nonzero. The inverse of N is:

$$N^{-1} = \frac{adj(N)}{|N|}$$

It is valid only when N is nonsingular. The adj(N) is the transposed cofactor matrix.

For the inconsistent linear system to have a unique least squares solution, the following equivalent conditions must be met.

a. The columns of A are linearly independent

b. The null space of A contains only the null vector

c. The matrix a is full range, R(A) = m

d. The normal matrix is nonsingular, det (ATA) ≠ 0

4. Development

The main objective of this work is to determine a mathematical model that establishes the relationship between the minimum visa time and the distance from the baseline for a recording interval of 5 seconds, in such a way that the required precision is maintained.

4.1 Equipment

The 16 Baselines were observed with the following equipment, software and logistics:

Equipment used

• Three GPS Receivers "SPECTRA PRECISION" GNSS, model R-6, dual frequency, of "Trimble Group"

• Dos Controladoras T41 W/SURVEY PRO GNSS, de “Trimble Group”

• Una Radio Modem ADL VANTAGE PRO KIT, 450-470 Mzs

• An RTK Radio Antenna

• Una batería PDLHPB - 33 AHR Battery with bag and cables, y cargador PDLHPB – 33 AHR Battery Charger.

• Two standard batteries with one charger

• Accessories: brackets, Tribrachs with optical plumb line, tribach adapters, wooden tripods, bipods, canes, etc.

• Dos Laptop Toshiba Core i7

• An HP Laser Printer, model Laser Jet 1020

• Dos GPS navigator Garmin ETREX 30

• A GPS navigator Garmin MONTERRA

• Two pickup trucks: Mitsubishi L 200 and Toyota Hilux

Software

• "Spectra Precision Survey Office v4.10 Complete 64"

4.2 GPS equipment configurations

• Dual frequency receivers, Geodesics

• Reading modes: Static.

• Dual frequency: L1 / L2 – L1 C/A – L2C _ L1P – L2 AC

• Radio: UHF 430-470 MHz

• Static differential mode accuracy: 0.003 m + 1 ppm RMS

• Number of channels:

Constelaciones: NAVSTAR GPS – GLONASS

• Number of visible satellites: > 5

• GDOP/PDOP: < 6

• Conversion data: Rinex

• SNR Mask: 15th

• Antenna type: SP80 UHF

• Antenna height (see Annex A.2.)

• Coordinate System: UTM

• Date: WGS 1984

• Geoid: EGM96 (Global)

• Zone: Depending on Location, it can be 17 South, 18 South or 19 South (see Annex A.3.

4.3 First stage: Reading and processing baselines

• A total of 16 baselines, independent, whose lengths vary from 733.62 m. (Baseline 1), to 95230.00 m. (Baseline 16), were observed at different time intervals, by the differential static method, all with a recording interval of 5 seconds.

• It is important to mention that all baselines belong to primary geodetic networks carried out for road studies throughout the national territory.

• The knowledge of the workplaces, location of the milestones and various technical details of them, are well known by the author of this thesis, since he has been directly responsible for its realization

• Therefore, all the vertices of the baselines to be studied are properly monumental.

• The baselines to be used have been grouped according to their length

Table 2. Road studies

PROJECT		DEPARTMENT	ERP	LOCALITY
P1	DEFINITIVE STUDY FOR THE IMPROVEMENT OF THE NEIGHBORHOOD ROAD PTE. ANGASMAYO – MILLPO; DV. HUARIPERJA - HUARIPERJA	AYACUCHO	AY01	ANGASMAYO
P2	IMPROVEMENT OF THE ROAD SHUPLUY-PRIMORPAMPA - BELLAVISTA - ANTA - SAN ISIDRO - PONCOS - KOCHAYOC - CHACLAHUAIN - ORATORIO - PAMPAMARCA - PUTACA	ANCASH	HUARAZ	SHUPLUY
P3	DEFINITIVE STUDY FOR THE CONSTRUCTION OF THE ROAD: CABALLOCOCHA - PALO SECO - BUEN SUCESO	LORETO - ITALY	PEBAS	CABALLOCOCHA
P4	IMPROVEMENT OF THE NEIGHBORHOOD ROAD EMP. CA-108 (PAMPA CHICA) – CAUDAY – DV. SAN ELIAS – EMP. CA-1653 (COIMA)	CAJAMARCA	CJ01	CAJABAMBA
P5	DEFINITIVE STUDY FOR THE "IMPROVEMENT OF THE	SAN MARTIN	SM01	YANTALO

	NEIGHBORHOOD ROAD EMP. SM-113 (YANTALO) - PUERTO LOS ANGELES (RIO MAYO)"			
P6	DEFINITIVE STUDY FOR THE IMPROVEMENT AND REHABILITATION OF THE CHAPINA - QUILLE NEIGHBORHOOD ROAD	CUSCO	CS02	CHAPINA
P7	DEFINITIVE STUDY FOR THE IMPROVEMENT AND REHABILITATION OF THE NEIGHBORHOOD ROAD EMP. R16 - INKACANCHA	CUSCO	CS01	INKACANCHA
P8	DEFINITIVE STUDY FOR THE "IMPROVEMENT OF THE NEIGHBORHOOD ROAD EMP. PE-5N – PURUS – RIOJA; EMP. PE-5N -SECTOR WINBA"	SAN MARTIN	SM01	RIOJA
P9	DEFINITIVE STUDY FOR THE "IMPROVEMENT OF THE NEIGHBORHOOD ROAD EMP. SM-116 – DV. CCNN EL NARANJAL – DV. LAMAS – CHIRAPA – PACCHILA – VENTANILLA – EMP. SM-682 (CACATACHI); DV. CCNN EL NARANJAL – EL NARANJAL – DV. SM-867 – EMP. DV. LAMAS; DV. SLATS – DV. SM-687 – DV. CCNN SHUKSHUYAKU – LAMAS; DV. CNN SHUKSHUYAKU – SHUKSHUYAKU"	SAN MARTIN	SM02	LAMAS
P10	DEFINITIVE STUDY FOR THE IMPROVEMENT OF THE NEIGHBORHOOD ROAD SOCOTA – SAN LUIS DE LUCMA – LA RAMADA	CAJAMARCA	CJ02	ERN

Source: Authors.

Table 3. Baselines to be used in this study

BASELINE	APPROXIMATE DISTANCE (m)	BASELINE		PROJECT	READINGS
		FROM: (FIXED COORDINATE)	A: FREE POINT		
1	1,10	GPS 01	T-14	INKACANCHA - CUSCO	12
2	2,95	GPS06	GPS04	ANGASMAYO - AYACUCHO	12
3	4,76	GPS01	GPS04	HORSE COCHA - LORETO	11
4	6,75	SM 01	GPS 01	YANTALO - SAN MARTIN	12
5	8,31	GPS08	GPS09	SHUPLUY-ANCASH	12
6	8,82	GPS03	T05	HORSE COCHA - LORETO	13
7	9,73	GPS08	T05	HORSE COCHA - LORETO	12
8	16,69	CJ02	GPS01 C	SOCOTA - CAJAMARCA	15
9	20,53	CJ02	GPS03 C	SOCOTA - CAJAMARCA	12
10	29,13	CJ02	GPS12 C	SOCOTA - CAJAMARCA	12
11	34,23	AY1	GPS10	ANGASMAYO - AYACUCHO	13
12	44,75	CS02	CCC04	CHAPINA-CUSCO	15
13	63,98	CJ01	GPS 07C	CAJABAMBA-CAJAMARCA	15
14	70,75	SM01	GPS10	LAMAS - SAN MARTIN	9
15	78,12	AM01	GPS10	SOCOTA - CAJAMARCA	16
16	95,23	CJ01	GPS10	SOCOTA - CAJAMARCA	16

Source: Authors.

Table 4. 16 baselines grouped by location

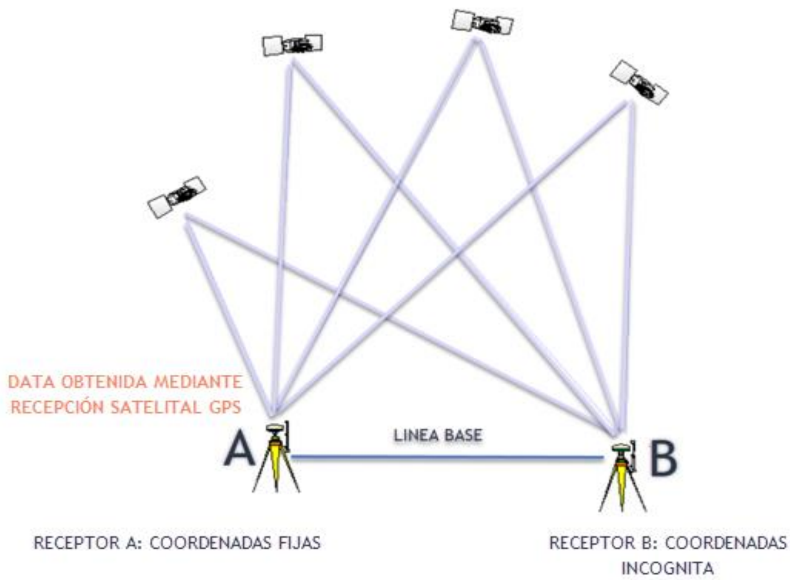
BASELINE	APPROXIMATE DISTANCE (M)	BASELINE		PROJECT	READINGS
		FROM: (FIXED COORDINATE)	A: FREE POINT		
1	1,10	GPS 01	T-14	INKACANCHA – CUSCO	12

12	44,75	CS02	CCC04	CHAPINA-CUSCO	15
2	2,95	GPS06	GPS04	ANGASMAYO - AYACUCHO	12
11	34,23	AY1	GPS10	ANGASMAYO - AYACUCHO	13
13	63,98	CJ01	GPS 07C	CAJABAMBA-CAJAMARCA	15
5	8,31	GPS08	GPS09	SHUPLUY-ANCASH	12
14	70,75	SM01	GPS10	LAMAS - SAN MARTIN	9
4	6,75	SM 01	GPS 01	YANTALO - SAN MARTIN	12
3	4,76	GPS01	GPS04	HORSE COCHA - LORETO	11
6	8,82	GPS03	T05	HORSE COCHA - LORETO	13
7	9,73	GPS08	T05	HORSE COCHA - LORETO	12
8	16,69	CJ02	GPS01C	SOCOTA - CAJAMARCA	15
9	20,53	CJ02	GPS03C	SOCOTA - CAJAMARCA	12
10	29,13	CJ02	GPS12C	SOCOTA - CAJAMARCA	12
15	78,12	AM01	GPS10	SOCOTA - CAJAMARCA	16
16	95,23	CJ01	GPS10	SOCOTA - CAJAMARCA	16

Source: Authors.

In the fieldwork: The measurement was started simultaneously at each end for an extended time (1.5 hours to 3 hours approximately).

Figure 2. Simultaneous Read model on a given baseline.



Source: Authors.

• After the observation session, these were stored in the internal memory of the receiver, these data are downloaded and processed on the computer with the specialized software "Spectra Precision Survey Office v4.10 Complete 64" performing the post process for the calculation of the coordinates in "B", for different time intervals.

• Once the post-process has been carried out, the report of the same is obtained: observation time, coordinates in "B" (East, North), standard deviation (East, North).

4.4 Results: First stage

4.4.1 Baseline 1

Table 5. Baseline 1 observations at different time intervals

BASELINE 1 MEASUREMENT AT DIFFERENT TIME INTERVALS					
	Duration h: min: s"	Norte UTM (m)	S NORTE	Este UTM (m)	S ESTE
1	00:02:00	8525453,135	0,002	207167,704	0,002
2	00:03:00	8525453,136	0,002	207167,704	0,002
3	00:05:00	8525453,138	0,003	207167,703	0,002
4	00:10:00	8525453,139	0,002	207167,704	0,002
5	00:15:00	8525453,142	0,002	207167,705	0,002
6	00:30:00	8525453,141	0,001	207167,706	0,001
7	01:00:00	8525453,141	0,001	207167,706	0,001
8	01:30:00	8525453,140	0,001	207167,706	0,001
9	02:00:00	8525453,140	0,001	207167,707	0,001
10	02:30:00	8525453,140	0,0004	207167,707	0,0004
11	03:00:00	8525453,140	0,0004	207167,707	0,0004
12	03:24:55	8525453,140	0,0004	207167,707	0,0004

Source: Authors.

4.4.2 Baseline 2

Table 6. Baseline 2 observations at different time intervals

BASELINE 2 MEASUREMENT AT DIFFERENT TIME INTERVALS					
	Duration h: min: s"	Norte UTM (m)	S NORTE	Este UTM (m)	S ESTE
1	00:06:05	8513788,733	0,004	-82878,142	0,005
2	00:10:05	8513788,732	0,004	-82878,141	0,004
3	00:12:05	8513788,731	0,003	-82878,139	0,004
4	00:13:05	8513788,731	0,003	-82878,139	0,004
5	00:14:05	8513788,731	0,003	-82878,139	0,004
6	00:18:05	8513788,731	0,003	-82878,137	0,003
7	00:28:05	8513788,734	0,003	-82878,143	0,003
8	00:43:05	8513788,734	0,002	-82878,139	0,003
9	00:58:05	8513788,733	0,002	-82878,141	0,002
10	01:13:05	8513788,734	0,002	-82878,140	0,002
11	01:28:05	8513788,733	0,001	-82878,140	0,002
12	01:58:05	8513788,732	0,001	-82878,141	0,001
13	02:28:10	8513788,732	0,001	-82878,141	0,001

Source: Authors.

4.4.3 Baseline 3

Table 7. Baseline 3 observations at different time intervals

BASELINE 3 MEASUREMENT AT DIFFERENT TIME INTERVALS					
	Duration h: min: s"	Norte UTM (m)	S NORTE	Este UTM (m)	S ESTE
1	00:09:00	9563595,371	0,004	327813,775	0,005
2	00:10:00	9563595,373	0,004	327813,776	0,005
3	00:13:00	9563595,373	0,004	327813,776	0,005

4	00:16:00	9563595,373	0,004	327813,776	0,005
5	00:18:00	9563595,374	0,004	327813,776	0,005
6	00:20:00	9563595,375	0,004	327813,774	0,005
7	00:25:00	9563595,377	0,004	327813,774	0,005
8	00:30:00	9563595,380	0,004	327813,774	0,005
9	00:45:00	9563595,385	0,003	327813,768	0,004
10	01:00:00	9563595,386	0,003	327813,765	0,004
11	01:15:00	9563595,385	0,003	327813,766	0,004
12	01:30:00	9563595,385	0,003	327813,767	0,004
13	02:00:00	9563595,384	0,003	327813,771	0,003
14	02:30:00	9563595,381	0,003	327813,771	0,003
15	03:030:00	9563595,380	0,003	327813,771	0,003

Source: Authors.

4.5 Second stage: Debugging errors

Having read and processed each baseline at different time intervals, we obtain a set of coordinates (North, East) with their respective standard deviation (σ NORTH, σ EAST), which may present some measurements with gross errors, for this we submit the set to a mathematical and statistical procedure, whose concepts are better explained in chapter II of this thesis.

The steps given below will be performed simultaneously the set of north coordinates and east coordinates.

- From the results of the first stage, for each baseline read at different time intervals, it is established as the a priori variance of the whole set equal to σ_0^2 σ_1^2 (variance of the first measured time interval).
- The matrix of weights [P]nxn is established, where "n" is the number of visualizations (measurements) of each baseline (Chapter 2.6).

$$P = \begin{bmatrix} \frac{\sigma_o^2}{\sigma_1^2} & 0 & \dots & \dots & 0 \\ 0 & \frac{\sigma_0^2}{\sigma_2^2} & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \frac{\sigma_o^2}{\sigma_n^2} \end{bmatrix}$$

- The following other matrices are established:

$[A]_{nx1}$ = Coefficient matrix where "n" is the number of readings per baseline

$[U]_{nx1}$ = Array of independent terms U

• The solution coordinates are obtained by least squares fitting (Chapter 2.7), as follows:

$$X = (A^T P\ A)^{-1} ATPU$$

• The residue matrix is found: [V] nx1 = AX-U

• The chi-square test statistic is found (Chapter 2.8).

$$\lambda^2 = \frac{V^T PV}{\sigma_o^2}$$

• The statistic is evaluated simultaneously, and must be in the acceptance zone, according to the critical values that depend on the degree of freedom and the level of significance α.λ^2

• If not, you have to delete a reading (n-1) and repeat the process.

• It is then evaluated simultaneously in Student's t-Reliability Test:

• $T_E = \frac{x-U}{S/\sqrt{n}}$

• Having to be in the acceptance zone of the Student's T-distribution plot, where its critical values also depend on the degree of freedom of the sample and the level of significance α.

• If not, you have to delete a reading (n-1) and repeat the process.

• Once the processes are completed, the acceptable readings would be obtained for each baseline, from which the one with the shortest reading time would be chosen, thus obtaining: the shortest visa time for a baseline of a certain length.

4.6 Second stage results

4.6.1 Baseline 1

• From the set of readings made to baseline 1 at different time intervals (Table N°9), the least squares adjustment is made, to later refine the set with the chi-square statistical test and the Student's T test.

• Obtaining the following refined data and the procedure followed to obtain it:

Table 8. Refined baseline 1

REFINED BASELINE **1**					
	Duration h: min: s"	**Norte UTM (m)**	**S NORTE**	**Este UTM (m)**	**S ESTE**
1	00:03:00	8525453,136	0,002	207167,704	0,002
2	00:10:00	8525453,139	0,002	207167,704	0,002

3	00:15:00	8525453,142	0,002	207167,705	0,002
4	00:30:00	8525453,141	0,001	207167,706	0,001
5	01:30:00	8525453,140	0,001	207167,706	0,001
6	02:00:00	8525453,140	0,001	207167,707	0,001
7	02:30:00	8525453,140	0,0004	207167,707	0,0004
8	03:00:00	8525453,140	0,0004	207167,707	0,0004
9	03:24:55	8525453,140	0,0004	207167,707	0,0004

Source: Authors.

- finding variance a prioriσ_0^2 :

VARIANZA A PRIORI NORTE		
Analizando Pesos		$P_i = \frac{K}{\sigma_i^2} = \frac{\sigma_0^2}{\sigma_i^2}$
Haciendo	P1=	1
	K=	σ_0^2
	σ_0^2 =	σ_1^2
VARIANZA A PRIORI	σ_0^2 =	0,000004

VARIANZA A PRIORI ESTE		
Analizando Pesos		$P_i = \frac{K}{\sigma_i^2} = \frac{\sigma_0^2}{\sigma_i^2}$
Haciendo	P1=	1
	K=	σ_0^2
	σ_0^2 =	σ_1^2
VARIANZA A PRIORI	σ_0^2 =	0,000004

• Matricialmente:

P NORTH =

NORTH OBSERVATION WEIGHT MATRIX								
1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	4	0	0	0	0	0
0	0	0	0	4	0	0	0	0
0	0	0	0	0	4	0	0	0
0	0	0	0	0	0	25	0	0
0	0	0	0	0	0	0	25	0
0	0	0	0	0	0	0	0	25

MATRIZ DE TÉRMINOS INDEPENDIENTES U	
U NORTE	U ESTE
8525453,136	207167,704
8525453,139	207167,704
8525453,142	207167,705
8525453,141	207167,706
8525453,140	207167,706
8525453,140	207167,707
8525453,140	207167,707
8525453,140	207167,707
8525453,140	207167,707

MATRIZ DE COEFICIENTES	
A =	1
	1
	1
	1
	1
	1
	1
	1
	1

P EAST=

EAST OBSERVATION WEIGHT MATRIX								
1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0

0	0	1	0	0	0	0	0	0
0	0	0	4	0	0	0	0	0
0	0	0	0	4	0	0	0	0
0	0	0	0	0	4	0	0	0
0	0	0	0	0	0	25	0	0
0	0	0	0	0	0	0	25	0
0	0	0	0	0	0	0	0	25

Solution by least squares:

COORDINATES AND NORTH BY LEAST SQUARES ADJUSTMENT		
	A^TP A=	90
	$(A^TP\ A)^{-1}$=	0,011111111
NORTH	A^TP U=	767290782,6
AND $_{NORTH}$=	$(A^TP\ A)^{-1}$ATP U=	8525453,14

X EAST COORDINATES **BY LEAST SQUARES ADJUSTMENT**		
	A^TP A=	90
	$(A^TP\ A)^{-1}$=	0,011111111
THIS	A^TP U=	18645093,61
X $_{EAST}$=	$(A^TP\ A)^{-1}$ATP U=	207167,7068

- Residue Matrix V:

RESIDUE MATRIX V	
V NORTE = IS$_{NORTH}$ -U	V IS =AX$_{IS}$ -U
V NORTH	**V IS**
0,0040	0,0028
0,0010	0,0028
-0,0020	0,0018
-0,0010	0,0008

0,0000	0,0008
0,0000	-0,0002
0,0000	-0,0002
0,0000	-0,0002
0,0000	-0,0002

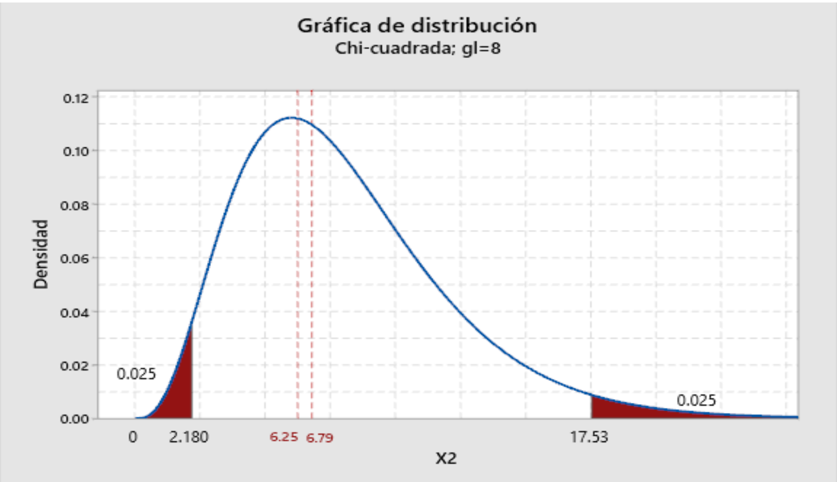
• Chi-square test:

TEST CHI CUADRADO		
GRADOS DE LIBERTAD	m=	9
	n=	1
	m-n=	8

ESTADISTICO DE PRUEBA		NORTE	ESTE
X^2=	V^TPV	2,49889E-05	2,71556E-05
	$\sigma 0^2$	0,000004	0,000004
	X^2=	**6,24722427**	**6,788888852**

VALORES CRÍTICOS EN LA DISTRIBUCIÓN CHI CUADRADO				
	MIN	X^2	MAX	RED
X^2 NORTE	2,180	6,24722427	17,535	CUMPLE
X^2 ESTE	2,180	6,788888852	17,535	CUMPLE
RESULTADO			RED EXITOSA	
Determinacion del X^2 MAX=			SE USA α/2	0,025
Determinacion del X^2 MIN=			SE USA 1- α/2	0,975

Figure 3. Chi square distribution distribution for purified baseline 1



Source: Authors.

- Test t de Student

TEST T STUDENT	
PROBABILIDAD	95%
GRADOS DE LIBERTAD	8
NIVEL DE SIGNIFICANCIA	5%

$$T_E = \frac{x - U}{S/\sqrt{n}}$$

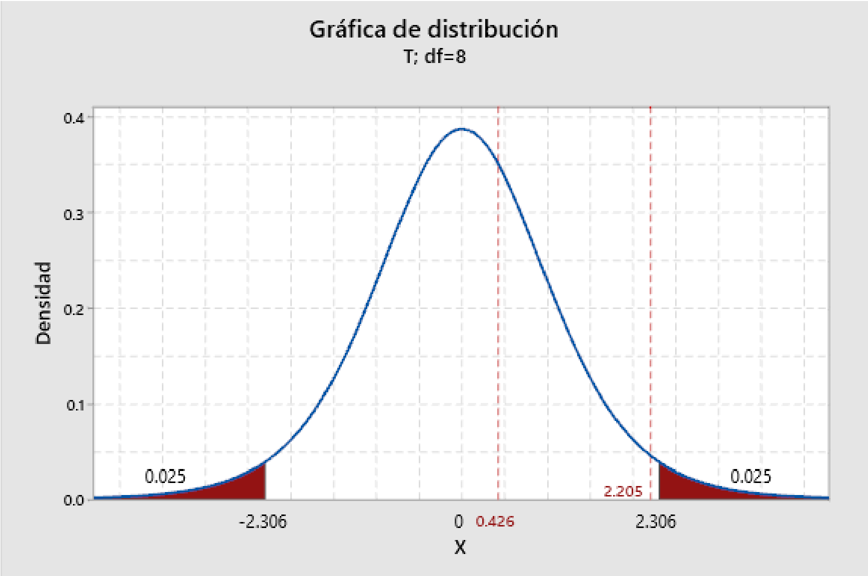
	X	U	S	n
NORTE	8525453,14	8525453,140	0,001641477	9
ESTE	207167,7068	207167,7059	0,001269296	9

T_E NORTE	X-U	0,00023	0,4264491
	$\frac{S}{\sqrt{n}}$	0,00055	
T_E ESTE	X-U	0,00093	2,2059481
	$\frac{S}{\sqrt{n}}$	0,00042	

ESTADISTICO T_E EN TABLA DE VALORES DE T DE STUDENT PARA 8 GRADOS DE LIBERTAD Y PROBABILIDAD 95%

	T 0.025	TE	T 0.025	RED
TE NORTE	-2,3060	0,426449134	2,3060	CUMPLE
TE ESTE	-2,3060	2,205948131	2,3060	CUMPLE
RESULTADO			RED EXITOSA	

Figure 4. Student's T-distribution for refined baseline 1



Source: Authors.

Figure 5. Minimum visa time for baseline 1 (Length =1100.00 m)

LÍNEA BASE 1 DEPURADA						
	Duración h: min: s"	Norte UTM (m)	σ NORTE	Este UTM (m)	σ ESTE
1	00:03:00	8525453,136	0,002	207167,704	0,002
2	00:10:00	8525453,139	0,002	207167,704	0,002
3	00:15:00	8525453,142	0,002	207167,705	0,002
4	00:30:00	8525453,141	0,001	207167,706	0,001
5	01:30:00	8525453,140	0,001	207167,706	0,001
6	02:00:00	8525453,140	0,001	207167,707	0,001
7	02:30:00	8525453,140	0,0004	207167,707	0,0004
8	03:00:00	8525453,140	0,0004	207167,707	0,0004
9	03:24:55	8525453,140	0,0004	207167,707	0,0004

Source: Authors.

5. Conclusions

The mathematical model between baseline and minimum visa time with high-precision GNSS technology for a 5-second recording interval is:

Y = 1.988 x + 2.448

Where:

Y= Minimum visa time for a given baseline (min).

X= Baseline length (km)

As a result of the mathematical model, it is possible to establish minimum observation times (expressed in whole numbers) for certain baseline lengths:

Table 9. Minimum observation times

DISTANCE (Km)	MINIMUM TIME (min)
1	5
2	7
5	13
10	23
20	43
30	63
40	82
50	102
60	122
70	142
80	162
90	182
100	202

Source: Authors.

This result is similar to the recommendations given by the company Leica Geosystems in its "Guide for measurements in static and fast static mode"

The representation of the dispersion: baseline vs observation time shows a linear trend, so a linear regression is arranged.

The Geodetic Technical Standard establishes a minimum observation time without considering the length of the baseline. In terms of productivity and accuracy it is preferable to make observations at shorter time intervals as long as the required precision is maintained.

In the process of debugging, initially we use the adjustment of least squares which presents us with an initial solution, which is called: solution by least squares, this same does not debug the errors of observation but shares it, so it is necessary to use the statistical techniques: chi square and t of Student, to eliminate gross errors of observation.

The Geodetic Technical Standard establishes the maximum separation of 100 km between base stations and the point to be established for geodetic points of order "C", so in this study our study interval is from 1.10 km to baseline 1 and 95.22 km

6. Recommendations

In any process of measuring a baseline, other processes intervene, which if neglected can ruin the observations in the field, generating losses of time and money, so it is recommended within these processes to follow protocols.

• Logistics: Related to equipment, calibration of high precision equipment, accessories, transportation, security.

• Analytical: Location of base points in order to meet an adequate observation, equipment configuration, usually directed by a specialist, engineer or technician

• Transfer of high precision equipment: protocols for moving, assembling and disassembling equipment, in order to avoid unnecessary damage to equipment.

• Processing: protocols where they avoid loss of geodetic data, includes backing up raw data and processed data.

Although the results of the present study are satisfactory and consistent, it is suggested that further studies be conducted using other brands of receptors, suggesting observations at larger and different lengths of baselines.

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