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Conversion Of Intuitionistic Fuzzy Muller Automata Into Construct Transition Intuitionistic Fuzzy Muller Automata

A. Uma∗ and S. Jayalakshmi †

∗Basic Engineering / Mathematics,
Government Polytechnic College. Tondiarpet,
Chennai, Tamilnadu, India.
† Department of Mathematics,
Annamalai University, Annamalainagar-608002, India.

Abstract

In this paper, Inspired by the intuitionistic fuzzy language as intuitionistic fuzzy Buchi automata, a new approach is proposed as the Muller automata into Transition Muller (Buchi) automata.

1 Introduction:

Automata an automata is a mathematical theory in investigates behavior, structure and their relationship to discrete systems. Directable automata were introduced by

P.H. Strake in[9] and J. Cerny in [5] and also definite automata by S.C. Kleene in[6]. Today, finite automata have many applications in plenty of areas of computer science such as databases, functional languages, bisimulations, and biology, for more information see[1,3]. Unlike automata and finite words, there are several types of automata on infinite words, differing in their acceptance conditions, most notably Buchi, Muller[7], Rabin.

The Muller condition explicitly lists the exact subsets of states that may be visited infinitely often along an accepting run. This is distinction to other types, such as Rabin, and Streett, which specify list of constraints on the subsets of states that are visited infinitely often. It is therefore reasonable to assume that Rabin and Street automata can be exponentially more succinct than Muller automata, which is indeed the case.[8] Using the notion of intuitionistic fuzzy sets [4] . The natural generalization of a fuzzy language as it is characterized by two functions expressing the degree of belongingness and the degree of non- belongingness. An intuitionistic fuzzy language is called intuitionistic fuzzy regular language, if the finite membership and non-membership values between [0,1][2].

The paper is organized as follows. In section 2, we recall the definitions of intuitionistic fuzzy automata. In section 3, we introduce the definition of intuitionistic fuzzy Muller automata and intuitionistic fuzzy transition automata with example. In section 4, we introduce the theorem in construction of transition intuitionistic fuzzy muller automata from intuitionistic fuzzy Muller automata with example.

2 Preliminaries:

Definition 2.1. Let a set 'E'be fixed. An intuitionistic fuzzy set 'A' in'E'is an object having the form $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in E\}$ where, the functions $\mu_A(x) \ : \ E \to [0, 1]$ and $\gamma_A(x) \ : \ E \to [0, 1]$ define the degree of membership and the degree of non membership of the element $x \in E$ totheset'A',

the subset of 'E'respectively, and for every$x \in E; 0 < \mu_A(x) + \gamma_A(x) \leq 1$.

Definition 2.2. An Intuitionistic fuzzy finite state machine is a triple $IM = (P, X, \delta)$

where,

P is a set of states.

X is an input alphabets.

$\delta = (\delta_1, \delta_2)$

Is an intuitionistic fuzzy subset of $P \times X \times P, \ni \forall q, p \in P, \forall x, y \in X$.

$\delta_1(p,\lambda,q) = \begin{cases} 1 \text{ if} q = p \\ 0 \text{ if } q \neq p \end{cases}$ $\delta_2(p,\lambda,q) = \begin{cases} 0 \text{ if } q = p \\ 1 \text{ if } q \neq p \end{cases}$

The function $\delta = (\delta_1, \delta_2)$is extended to the set of words $w \in X^*$as follows.

$$\delta_1(p,w,q) = \max\{\min(\delta_1^*(p,w_1,r), \delta_1(r,a,q))/w = w_1 a \forall w_1 \in X^*, a \in X\}$$

$$\delta_2(p,w,q) = \min\{\max(\delta_2^*(p,w_1,r), \delta_2(r,a,q))/w = w_1 a \forall w_1 \in X^*, a \in X\}.$$

Definition 2.3. An Intuitionistic fuzzy automata is a five tuple $IM = (P, X, \delta, p_0, F)$where,

P is a finite non-empty set of states,

X is a finite non-empty set of inputs,

$\delta : P \times X \times P \to [0,1]$is called the intuitionistic fuzzy transition function.

p_0 : is called the initial state and

F : is called the set of final states.

Definition 2.4. Let $IM = (P, X, \delta, p_0, F)$ be an intuitionistic fuzzy automaton. Then the language $L(IM) = \{(x,(\mu,\gamma))|x \in X^*, \delta(p_0, w, q_f) = (\mu,\gamma), q_f \in F\}$ is called the intuitionistic fuzzy regular language accepted by intuitionistic fuzzy automat on IM.

Definition 2.5. An intuitionistic fuzzy Buchi automaton is a 5-tuple $K = (P, N, q, S, \mathfrak{A})$, where

P is the finite set of states.

N is the finite set of input alphabets.

$q: P \times N \times P \to (\delta_1, \delta_2), 0 < \delta_1 + \delta_2 \leq 1$is the intuitionistic fuzzy transition function.

where,

$\delta_1(q(p_i, w, p_j)) = \vee \{\delta_1(p_i, w_1, p_k) \wedge \delta_1(p_k, a, p_j)\}$ where, $w = w_1 a_1$for all $p_k \in P$

$\delta_2(q(p_i, w, p_j)) = \wedge \{\delta_2(p_i, w_1, p_k) \vee \delta_2(p_k, a, p_j)\}$ where, $w = w_1 a_1$for all $p_k \in P$.

$S : \to (\delta_1, \delta_2)$, where$0 < \delta_1 + \delta_2 \leq 1$ is the set of intuitionistic fuzzy initial states.

$\mathfrak{A} : P \to (\delta_1, \delta_2)$, where $0 < \delta_1 + \delta_2 \leq 1$ is the set of intuitionistic fuzzy final states. A run inintuitionistic fuzzy Buchi automata is successful if it visit A infinitely often, that is $\inf(\mathfrak{R}) \neq \varphi$

The weight of the accepted word $w(\delta_1, \delta_2)$ is calculated as follows.

$$w(\delta_1) = \vee \{\wedge \{\{S(p_1)\} \cup \{q(p_i, w, p_j) | i, j \geq 1\} \cup \{\mathfrak{A}(t)/t \in \inf(\mathfrak{R}) \cap \mathfrak{A}$$

$$w(\delta_2) = \wedge \{\vee \{\{S(p_1)\} \cup \{q(p_i, w, p_j) | i, j \geq 1\} \cup \{\mathfrak{A}(t)/t \in \inf(\mathfrak{R}) \cap \mathfrak{A}$$

3 .INTUITIONISTIC FUZZY MULLERAUTOMATA

3.1 Definition

An intuitionistic fuzzy Muller Automata is a 5-tuple

$V = (P, I, \delta, p, \mathfrak{M})$, where

P is the finite set of states.

I is the finite set of input alphabets.

$\delta : P \times I \times P \to (d_1, d_2)$, where $0 < d_1 + d_2 \leq 1$ is the Intuitionistic fuzzy transition

function.

$p : \to (d_1, d_2)$, where $0 < d_1 + d_2 \leq 1$ is the set of Initial States.

$\mathfrak{M}$ is a set of Intuitionistic fuzzy subsets of P,that is $\mathfrak{M} \subset F(p)$.

A run in successful if the set of infinitely often states is a member of $\mathfrak{M}$, that is inf (R) $\in \mathfrak{M}$

The weight of the accepted word $w(d_1, d_2)$ is calculated as follows.

$$w(d_1) = max \{ min \{\{P(p_1)\} \cup \{\delta(p_i, w, p_j) | i, j \geq 1\} \cup \mathfrak{M}(t) | t \in inf\ (R)$$

$$w(d_2) = min \{ max \{\{P(p_1)\} \cup \{\delta(p_i, w, p_j) | i, j \geq 1\} \cup \mathfrak{M}(t) | t \in inf\ (R)$$

An intuitionistic fuzzy Muller automaton is said to be deterministic intuitionistic fuzzy Muller automaton if it has deterministic transition. Since, every word in IL (V) is the label of exactly one run, the weight of the accepted word $w(d_1, d_2)$ in a deterministic intuitionistic fuzzy Muller automaton is calculated as follows

$w(d_1) = min\{\{I(p_1)\} \cup \{f(p_i, w_j, p_k)|i,j,k \geq 1\} \cup \{\mathfrak{M}(t)|r \in inf\ (R)\}\}$

$w(d_2) = max\{\{I(p_1)\} \cup \{f(p_i, w_j, p_k)|i,j,k \geq 1\} \cup \{\mathfrak{M}(t)|r \in inf\ (R)\}\}$

Theorem 3.1. An intuitionistic fuzzy language recognized by an intuitionistic fuzzy Muller automaton is also recognized by a complete intuitionistic fuzzy Muller automaton.

Proof Let V $= (P, I, \delta, p_1, M)$ be an intuitionistic fuzzy Muller automaton. Construct a complete intuitionistic fuzzy Muller automaton $V = (P', I, \delta', I, \mathfrak{M})$

where $P' = P \cup \{e\}, e$ is the new state and define

$\delta' : P' \times I \times P' \to [1,0]$ as follows: $\delta' = \delta_1 \cup \delta_2$ for all $a \in I$

$\delta_1(d_1)(e, a, e) = 0$

$\delta_1(d_2)(e, a, e) = 1$ for all $p, t \in P'$ and $a \in I$

$$\delta_2(d_1)(p, a, t) = \begin{cases} \delta(p, a, t), & if\ \delta(p, a, t) \neq 0; \\ 0, & if\ \delta(p, a, t) = 0. \end{cases}$$

$$\delta_2(d_2)(p, a, t) = \begin{cases} \delta(p, a, t), & if\ \delta(p, a, t) \neq 0; \\ 1, & if\ \delta(p, a, t) = 0. \end{cases}$$

since all the states in P is a state in P', every transition in V is a transition in V' and the intuitionistic fuzzy initial and intuitionistic fuzzy final states are same as in V. So every intuitionistic fuzzy $w(d_1, d_2)$-word accepted by V is also accepted by V'. Therefore, for every intuitionistic fuzzy Muller automaton V there exists a complete intuitionistic fuzzy Muller automaton V' such that V and V' recognize the same intuitionistic fuzzy $w(d_1, d_2)$-language.

4 . Intuitionistic fuzzy transition automata.

4.1 Definition

An intuitionistic fuzzy transition Buchi automaton is a 5-tuple V= $(P, I, p, \delta, \mathfrak{A})$, where

P is the finite set of states.

I is the finite set of input alphabets.

p : P $\to (d_1, d_2)$, where $0 < d_1 + d_2 \leq 1$ is the set of intuitionistic fuzzy initial states.

$\delta : P \times I \times P \to (d_1, d_2), 0 < d_1 + d_2 \leq 1$ is the intuitionistic fuzzy transition function.

where,

$d_1(p(p_i, w, p_j)) = \vee \{d_1(p_i, w_1, p_k) \wedge d_1(p_k, a, p_j)\}$ where, $w = w_1 a_1$ for all $p_k \in P d_2(p(p_i, w, p_j)) = \wedge \{d_2(p_i, w_1, p_k) \vee d_2(p_k, a, p_j)\}$ where, $w = w_1 a_1$ for all $p_k \in P$

$\mathfrak{A} \subseteq \delta(d_1, d_2)$, where $0 < d_1 + d_2 \leq 1$ is the set of intuitionistic fuzzy final transitions.

4.2 Definition

An intuitionistic fuzzy transition Muller Automata is a 5-tuple

$V = (P, I, \delta, p, \mathfrak{M})$, where

P is the finite set of states.

I is the finite set of input alphabets.

$\delta : P \times I \times P \to (d_1, d_2)$, where $0 < d_1 + d_2 \leq 1$ is the Intuitionistic fuzzy transition function.

$p : \to (d_1, d_2)$, where $0 < d_1 + d_2 \leq 1$ is the set of Initial States.

$\mathfrak{M}$ is a set of Intuitionistic fuzzy subsets of δ.

Theorem 4.1. An intuitionistic fuzzy language recognized by a intuitionistic fuzzy deterministic Muller(Buchi) automaton is also recognized by a transition intuitionistic fuzzy deterministic Muller(Buchi) automaton.

Proof. Let V $= (P, I, \delta, p, \mathfrak{A})$ be a deterministic fuzzy Muller (Buchi) automaton. Construct a transition deterministic fuzzy Muller (Buchi) automaton

V'= $(\mathrm{P}', \mathrm{I}, \delta', \mathrm{q}, \mathfrak{A}')$, where

$\mathrm{P}' = \delta \cup \{\mathrm{q}\}$

I is the finite set of input alphabets.

$$\delta' = \{(\mathrm{p}, \mathrm{a}, (\mathrm{p}, \mathrm{a}, \mathrm{t}, \mathrm{c}), \mathrm{c}) | (\mathrm{p}, \mathrm{a}, \mathrm{t}, \mathrm{c}) \in \delta\} \cup \{((\mathrm{t}, \mathrm{a}, \mathrm{t}', \mathrm{c}), \mathrm{b}, (\mathrm{t}', \mathrm{b}, \mathrm{td}), \mathrm{d}) | (\mathrm{t}, \mathrm{a}, \mathrm{t}', \mathrm{c}) \& (\mathrm{t}, \mathrm{b}, \mathrm{td}) \in \delta\}$$

$\mathrm{q}: \mathrm{p} \to \mathrm{F}$ is a new state.

$\mathfrak{A}' =$ A. Let $\mathrm{w}(\mathrm{d}_1, \mathrm{d}_2) \in$IL (V) , then $\mathrm{w}(\mathrm{d}_1, \mathrm{d}_2) = \mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3, \cdots$ is the label of a run $\mathrm{R} = \mathrm{p}_1 \mathrm{p}_\mathrm{i} \mathrm{p}_\mathrm{j} \mathrm{p}_\mathrm{k} \cdots$ in V, there is a corresponding run

$R' = p(p_1, a_1, p_i, c_i)(p_i, a_2, p_j, c_j)(p_j, a_3, p_k, c_k)\cdots$ of $w(d_1, d_2)$in V. From the construction the weight of the accepted word $w(d_1, d_2)$ in V is same as in V. Hence $(d_1, d_2) \in IL(V')$. Conversely, let $(d_1, d_2) \in IL(V')$, then $w(d_1, d_2)$ is the label of a run R' in V'. $w(d_1, d_2)$ visits the transition (p_i, a, p_j, c_j) in V if and only if R' visits the state (p_i, a, p_j, c_j) and from the construction the weight of the accepted word $w(d_1, d_2)$ in V is same as in V'.

Hence $(d_1, d_2) \in IL(V)$.

Illustrative Example

Consider the intuitionistic fuzzy deterministic Muller automaton Consider the fuzzy deterministic Muller automaton $V = (P, I, \delta, q, \mathfrak{M})$,where

$P = \{p, t\}$,

$I = \{a, b\}$,

$q = \{p(1,0)\}$

$\mathfrak{M} = \{t(1,0)\}$

The transition diagram of this intuitionistic fuzzy automaton The intuitionistic fuzzy language accepted $IL(V) = \{w(d_1, d_2)\} \in (b + a)^* a^{(j)}, (0.2,0.5)$ }

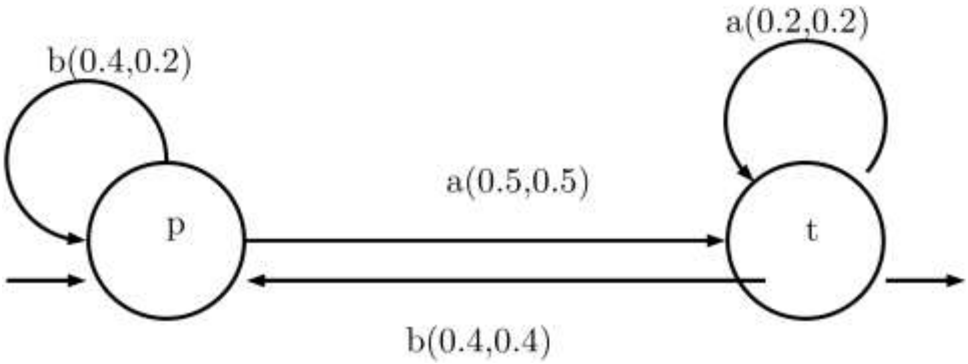

Figure 4.1 : Intuitionistic fuzzy automata

By the above construction procedure,

we obtain a transition intuitionistic fuzzy deterministic Muller automaton

V'= $(P, I, \delta, p, \mathfrak{M})$, where

$P' = \{p, (p, b, p), (p, a, t), (t, b, p), (t, a, t)\}$

$I = (p(1,0))$,

$\delta'(q, b, (p, b, p)) = (0.4,0.2)$

$\delta'(p, a, (s, a, t)) = (0.5, 0.5)$

$\delta'((p, b, p), b, (p, b, p)) = (0.4, 0.2)$

$\delta'((p, b, p), a, (p, a, t)) = (0.5, 0.5)$

$\delta'((p, a, t), b, (t, b, p)) = (0.4, 0.4)$

$\delta'((p, a, t), a, (t, a, t)) = (0.2, 0.2)$

$\delta'((t, b, p), b, (p, b, p)) = (0.4, 0.2)$

$\delta'((t, b, p), a, (p, a, t)) = (0.5, 0.5)$

$\delta'((t, a, t), b, (t, b, p)) = (0.4, 0.4)$

$\delta'((t, a, t), a, (t, a, t)) = (0.2, 0.2)$

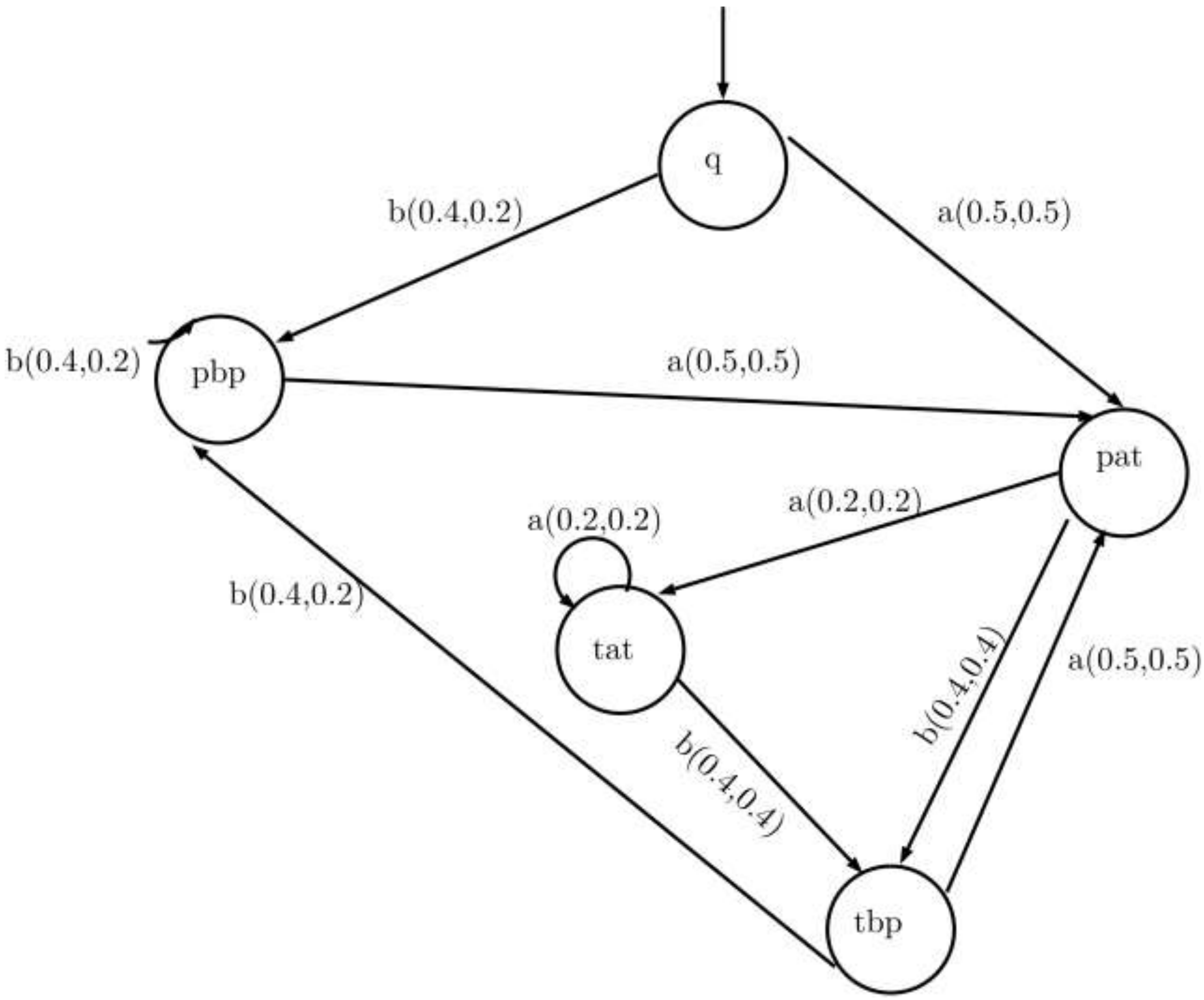

Figure 4.2: Transition Intuitionistic Fuzzy Muller Automata

This example illustrates that a intuitionistic fuzzy language recognized by a intuitionistic fuzzy deterministic Muller automata is also recognized by a transition intuitionistic fuzzy deterministic Muller automata.

5. Conclusions

This paper generalize intuitionistic fuzzy muller automata into intuitionistic fuzzy Buchi automata and also derived the definition of intuitionistic fuzzy Buchi transition and Muller transition automata. then, proved the theorem in consider the intuitionistic muller automata to construct transition intuitionistic muller automata with example.

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