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Normal Spaces Associated With P*Gb-Open Sets

Aruna Glory Sudha. I[*] , Dr. S. Zion Chella Ruth[†]

*Aruna Glory Sudha. I, Research Scholar, Registration Number: 21212152092002 , Department of Mathematics, Nazareth Margoschis College at Pillaiyanmanai, Nazareth, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, Tamilnadu, India. e-mail: sudhaagers@gmail.com.

† Dr. S. Zion Chella Ruth, Assistant Professor, Department of Mathematics, Nazareth Margoschis College at Pillaiyanmanai, Nazareth, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, Tamilnadu, India. e-mail: ruthalwin@gmail.com.

Abstract

We present and explore pre-star generalized b-normal spaces and their stronger and weaker form spaces utilizing the paradigm of pre-star generalized b-closed and pre-star generalized b-open sets.

Keywords: p*gb-open, p*gb-closed, p*gb-normal, strongly p*gb-normal, weakly p*gb-normal.

1. INTRODUCTION

Pre*-closed sets were presented and some of their characteristics were studied by T. Selvi and A. Punitha Dharani [3] in 2012. Pre*-generalized b-closed and b-open set characteristics are provided in [4].The pre* generalized b-normal and strongly p*gb-normal and weakly p*gb-normal spaces that we describe and investigate in this work make use of p*gb-open and p*gb-closed sets, respectively.

2. PRELIMINARIES

Definition 2.1.[1] In X, A subset M is called

(i)b-open if M⊆Int (Cl(M))∪Cl (Int(M))

(ii)b-closed if Int (Cl(M))∩Cl (Int(M)) ⊆M.

Definition 2.2.[1] b-closure of A, denoted by bCl(A)=⋂ {H: A⊆H and H is b-closed}.

Definition 2.3. [3] A subset M of the space X is called

(i) pre*-open if M⊆int*(Cl(M))

(ii) pre*-closed if Cl*(Int(M))⊆M.

Definition 2.4.[4] A pre* generalized b-closed set (briefly, p*gb-closed) is a subset A of a Space (X, τ) if bCl(A)⊆U, whenever A⊆U, U is pre*-open in (X, τ).

Lemma 2.5.[4] For a topological space (X,τ), Every open set is p*gb-open.

Lemma 2.6. [4]

(a) Arbitrary intersection of p*gb-closed sets is p*gb-closed.
(b) Arbitrary union of p*gb-open sets is p*gb-open.

Remark 2.7.[4]

(a) The union of p*gb-closed sets need not be a p*gb-closed set.
(b) The intersection of p*gb-open sets is p*gb-open.

Definition 2.8.[4] Let X be a topological space and let xof X A subset N of X is said to be a p*gb-neighbourhood (shortly, p*gb-nbhd) of x if there exists a p*gb-open set U such that x∈U⊆N.

Theorem 2.9.[4] Every nbhd N of x∈X is a p*gb-nbhd of x.

Definition 2.10.[4] Let A be a subset of a topological space (X, τ). Then the union of all p*gb-open sets contained in A is called the p*gb-interior of A and it is denoted by p*gbInt(A). That is, p*gbInt(A)=∪{V:V⊆A and V∈p*gb-O(X)}.

Theorem 2.11.[4] Let A be a subset of a topological space (X, τ). Then

(a) p*gbInt(A) is the largest p*gb-open set contained in A.

(b) A is p*gb-open if and only if p*gbInt(A)=A.
(c) p*gbInt(ϕ)= ϕ and p*gbInt(X)=X.
(d) If $A\subseteq B$, then p*gbInt(A)$\subseteq$p*gbInt(B).
(e) p*gbInt(p*gbInt(A))=p*gbInt(A).

Definition 2.12.[4] Let A be a subset of a topological space (X, τ). Then the intersection of all p*gb-closed sets in X containing A is called the p*gb-closure of A and it is denoted by p*gbCl(A). That is, p*gbCl(A)=$\cap\{F: A\subseteq F$ and $F\in$p*gb-C(X)$\}$. The intersection of the p*gb-closed set is p*gb-closed, then p*gbCl(A) is p*gb-closed.

Theorem 2.13.[4] Let A be a subset of a topological space (X, τ). Then

(a) p*gbCl(A) is the smallest p*gb-closed set containing A.
(b) A is p*gb-closed if and only if p*gbCl(A)=A.
(c) p*gbCl(ϕ)= $\phi$and p*gbCl(X)=X.
(d) If $A\subseteq B$, then p*gbCl(A)$\subseteq$p*gbCl(B).
(e) p*gbCl(p*gbCl(A))=p*gbCl(A).

Definition 2.14[5]. A topological space X is quasi-H-closed if every open cover has a finite proximate subcover. That is, every open cover has a finite subfamily whose closures cover the space.

Definition 2.15[5]. A topological space (X,τ) is said to be **normal** if, for disjoint closed sets A and B, there exist disjoint open sets U and V such that $A\subseteq U$, $B\subseteq V$.

3. p*gb-normal spaces

Definition 3.1. A topological space (X, τ) is said to be p*gb-normal if for any two disjoint p*gb-closed sets A and B, there exist disjoint p*gb-open sets U and V such that $A\subseteq U$ and $B\subseteq V$.

Theorem 3.2. In a topological space X, the following are equivalent:
(a) X is p*gb-normal.
(b) For every p*gb-closed set A in X and every p*gb-open set U containing A, there exists a p*gb- open set V containing A such that p*gbcl(V)$\subseteq$U.

(c) For each pair of disjoint p*gb-closed sets A and B in X, there exists a p*gb-open set U containing A such that p*gbcl(U)∩B=ϕ.

(d) For each pair of disjoint p*gb-closed sets A and B in X, there exist p*gb-open sets U and V containing A and B respectively such that p*gbcl(U)∩p*gbcl(V)=ϕ.

Proof: (a)→(b): Let U be a p*gb-open set containing the p*gb-closed set A. Then B=X\U is a p*gb-closed set disjoint from A. Since X is p*gb-normal, there exist disjoint p*gb-open sets V and W containing A and B respectively. Then p*gbcl(V) is disjoint from B, since if y∈B, the set W is a p*gb-open set containing y disjoint from V. Hence p*gbcl(V)⊆U.

(b)→(c): Let A and B be disjoint p*gb-closed sets in X. Then X\B is a p*gb-open set containing A. By (ii), there exists a p*gb-open set U containing A such that p*gbcl(U) X\B. Hence p*gbcl(U)∩B=ϕ. This proves (c).

(c)→(d): Let A and B be disjoint p*gb-closed sets in X. Then, by (iii), there exists a p*gb-open set U containing A such that p*gbcl(U)∩B=ϕ. Since p*gbcl(U) is p*gb-closed**,** B and p*gbcl(U) are disjoint p*gb-closed sets in X. Again by (iii), there exists a p*gb-open set V containing B such that p*gbcl(U)∩p*gbcl(V)=ϕ. This proves (d).

(d)→(a): Let A and B be the disjoint p*gb-closed sets in X. By (iv), there exist p*gb-open sets U and V containing A and B respectively such that p*gbcl(U)∩p*gbcl(V)=ϕ. Since U∩V⊆p*gbcl(U)∩p*gbcl(V), U and V are disjoint p*gb-open sets containing A and B respectively. Thus, X is p*gb-normal.

Theorem 3.3. A space (X,τ) is p*gb-normal if and only if for every p*gb-closed set F and p*gb-open set G containing F, there exists a p*gb-open set V such that F⊆V⊆p*gbcl(V)⊆G.

Proof: Let (X,τ) be p*gb-normal. Let F be a p*gb-closed set and let G be a p*gb-open set containing F. Then F and X\G are disjoint p*gb-closed sets. Since X is p*gb-normal, there exist disjoint p*gb-open sets V_1 and V_2 such that F⊆V_1 and X\G⊆V_2. Thus F⊆V_1⊆X\$V_2$⊆G. Since X\$V_2$ is p*gb-closed, so p*gbcl(V_1)⊆p*gbcl(X\$V_2$) =X\$V_2$⊆G. Take V=$V_1$. This implies that F⊆V⊆p*gbcl(V)⊆G. Conversely suppose the condition holds. Let H_1 and H_2 be two disjoint p*gb-closed sets in X.

Then $X \backslash H_2$ is an p*gb-open set containing H_1. By assumption, there exists a p*gb-open set V such that $H_1 \subseteq V \subseteq$ p*gbcl (V)$\subseteq X \backslash H_2$. Since V is p*gb-open and p*gbcl(V) is p*gb-closed. Then X\p*gbcl(V) is p*gb-open. Now p*gbcl(V) $\subseteq X \backslash H_2$ implies that $H_2 \subseteq$ X\p*gbcl(V). Also, V$\cap$(X\p*gbcl(V) $\subseteq$p*gbcl(V)$\cap$(X\p*gbcl(V)) =ϕ. That is V and X\p*gbcl(V) are disjoint p*gb-open sets containing H_1 and H_2 respectively. This shows that (X,τ) is p*gb-normal.

Theorem 3.4. For a space X, then the following are equivalent:

(a) X is p*gb-normal.

(b) For any two p*gb-open sets U and V whose union is X, there exist p*gb-closed subsets A of U and B of V whose union is also X.

Proof: (a)→(b): Let U and V be two p*gb-open sets in a p*gb-normal space X such that X=U∪V. Then X\U, X\V are disjoint p*gb-closed sets. Since X is p*gb-normal, then there exist disjoint p*gb-open sets G_1 and G_2 such that X\U⊆G1 and X\V⊆G_2. Let A=X\$G_1$ and B=X\$G_2$. Then A and B are p*gb-closed subsets of U and V respectively such that A∪B=X. This proves (b).

(b)→(a): Let A and B be disjoint p*gb-closed sets in X. Then X\A and X\B are p*gb-open sets whose union is X. By (ii), there exists p*gb-closed sets F_1 and F_2 such that $F_1 \subseteq X \backslash A$, $F_2 \subseteq X \backslash B$ and $F_1 \cup F_2 = X$. Then $X \backslash F_1$ and $X \backslash F_2$ are disjoint p*gb-open sets containing A and B respectively. Therefore, X is p*gb-normal.

Theorem 3.5. Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a function.

(a) If f is injective, p*gb-irresolute, p*gb-open and X is p*gb-normal then Y is p*gb-normal.

(b) If f is p*gb-irresolute, p*gb-closed and Y is p*gb-normal then X is p*gb-normal.

Proof: (a) Suppose X is p*gb-normal. Let A and B be disjoint p*gb-closed sets in Y. Since f is p*gb-irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are p*gb-closed in X. Since X is p*gb-normal, there exist disjoint p*gb-open sets U and V in X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Now $f^{-1}(A) \subseteq U \Rightarrow A \subseteq f(U)$ and $f^{-1}(B) \subseteq V \Rightarrow B \subseteq f(V)$. Since f is a p*gb-open map, f(U) and f(V) are p*gb-open in Y. Also, $U \cap V = \phi \Rightarrow f(U \cap V) = \phi$ and f is injective, then $f(U) \cap f(V) = \phi$. Thus f(U) and f(V) are disjoint p*gb-open sets in Y containing A and B respectively. Thus, Y is p*gb-normal.

(b)Suppose Y is p*gb-normal. Let A and B be disjoint p*gb-closed sets in X. Since f is p*gb-irresolute and p*gb-closed, f(A) and f(B) are p*gb-closed in Y. Since Y is p*gb-normal, there exist disjoint p*gb-open sets U and V in Y such that f(A)$\subseteq$U and f(B)$\subseteq$V. That is A$\subseteq f^{-1}$(U) and B$\subseteq f^{-1}$(V). Since f is p*gb-irresolute, f^{-1}(U) and f^{-1}(V) are disjoint p*gb-open such that A$\subseteq f^{-1}$(U) and B$\subseteq f^{-1}$(V). Thus, X is p*gb-normal.

Theorem 3.6. If given a pair of disjoint p*gb-closed sets A, B of X, there is p*gb-continuous function f:X→ [0,1] such that f(A)= {0} and f(B)= {1}, then (X,τ) is p*gb-normal.

Proof: Let (X,τ) be a topological space. Suppose for any pair of disjoint p*gb-closed sets A, B in X, there exists a p*gb-continuous map f:X → [0,1] such that f(A)= {0} and f(B)= {1}. Let E and F be disjoint p*gb-closed sets in X. Lt a, b$\in$[0, 1]at a$\leq$b. Take G= [0, a) and H=(b,1]. Then G and H are disjoint open sets in [0, 1]. Since f is p*gb-continuous, f^{-1}(G) and f^{-1}(H) are p*gb-open in X. By our assumption, f(E)= {0} and f(F)= {1}. Now f(E)= {0} implies f^{-1}(f(E))$\subseteq f^{-1}$({0})$\Rightarrow$E$\subseteq f^{-1}$(f(E))$\subseteq f^{-1}$({0})$\Rightarrow$E$\subseteq f^{-1}$({0}). Similarly, F$\subseteq f^{-1}$({1}). Evidently, {0}$\subseteq$[0, a)$\Rightarrow$ f^{-1}({0})$\subseteq f^{-1}$([0, a)). This implies that E$\subseteq$ f^{-1}({0})$\subseteq f^{-1}$([0, a)) = f^{-1}(G). Also {1}$\subseteq$(b, 1]$\Rightarrow$ f^{-1}({1})$\subseteq$ f^{-1}((b, 1]) which implies F$\subseteq f^{-1}$({1})$\subseteq$ f^{-1}((b, 1]) = f^{-1}(H). Fuhrer, f^{-1}(G) $\cap$ f^{-1}(H)= f^{-1}(G$\cap$ H f^{-1}(ϕ)=ϕ. So, we have a pair of disjoint p*gb-open sets, f^{-1}(G), f^{-1}(H)$\subseteq$X such that E$\subseteq$ F$\subseteq f^{-1}$(H). This proves that (X,τ) is p*gb-normal.

Theorem 3.7. If f is a p*gb-continuous and closed injection of a topological space X into a normal space Y and if every p*gb-closed set in X is closed, then X is p*gb-normal.

Proof: Let A and B be disjoint p*gb-closed sets in X. By assumption, A and B are closed in X. Then f(A) and f(B) are disjoint closed sets in Y. Since Y is normal, there exist disjoint open sets V_1 and V_2 in Y such that f(A)⊆ $_1$ and f(B)⊆V_2. Then f^{-1}(V_1) and f^{-1}(V_2) are disjoint p*gb-open sets in X containing A and B respectively. Hence X is p*gb-normal.

Theorem 3.8. If f is a continuous p*gb-open bijection of a normal space X into a space Y and if every p*gb-closed set in Y is closed, then Y is p*gb-normal.

Proof: Let A and B be p*gb-closed set in Y. Then by assumption, B is closed in Y. Since f is a continuous bijection, f^{-1}(A) and f^{-1}(B) is a closed set in X. Since X is normal there exist disjoint open sets U_1 and U_2 in X such that $f^{-1}(A)\subseteq U_1$ and $f^{-1}(B)\subseteq U_2$. Since f is p*gb-open, f(U_1) and f (U_2) are disjoint p*gb-open sets in Y containing A and B respectively. Hence Y is p*gb-normal.

4. Strongly p*gb-normal spaces

Definition 4.1. A topological space (X, τ) is said to be strongly p*gb-normal if for any two disjoint p*gb-closed sets A and B, there exist disjoint open sets U and V such that $A\subseteq U$ and $B\subseteq V$.

Proposition 4.2. Every strongly p*gb-normal space is p*gb-normal.

Proof: Suppose X is strongly p*gb-normal. Let F and G be disjoint p*gb-closed sets. Since X is strongly p*gb-normal, there exist disjoint open sets U and V such that equal U and $F\subseteq V$. Since every open set is p*gb-open, U and V are p*gb-open sets with the condition $G\subseteq U$ and $F\subseteq V$. This implies that X is p*gb-normal.

Theorem 4.3. In a topological space X, the following are equivalent:

a) X is strongly p*gb-normal.
b) For every p*gb-closed set A in X and every p*gb-open set U containing A, there exists an open set V containing A such that $cl(V)\subseteq U$.
c) For each pair of disjoint p*gb-closed sets A and B in X, there exists an open set U containing A such that $cl(U)\cap B=\phi$.
d) For each pair of disjoint p*gb-closed sets A and B in X, there exist open sets U and V containing A and B respectively such that $cl(U)\cap cl(V)=\phi$.

Proof: (a)→(b): Let U be a p*gb-open set containing the p*gb-closed set A. Then $B=X\setminus U$ is a p*gb-closed set disjoint from A. Since X is strongly p*gb-normal, there exist disjoint open sets V and W containing A and B respectively. Then cl(V) is disjoint from B, since if $y\in B$, the set W is an open set containing y disjoint from V. Hence $cl(V)\subseteq U$.

(b)→(c): Let A and B be disjoint p*gb-closed sets in X. Then X\B is a p*gb-open set containing A. By (ii), there exists an open set U containing A such that cl(U)⊆X\B. Hence cl(U)∩B=ϕ. This proves (c).

$(c) \rightarrow (d)$: Let A and B be disjoint p*gb-closed sets in X. Then, by (iii), there exists an open set U containing A such that cl(U)∩B=ϕ. Since cl(U) is p*gb-closed**,** B and cl(U) are disjoint p*gb-closed sets in X. Again by (iii), there exists an open set V containing B such that cl(U)∩cl(V)=ϕ. This proves (d).

(d)→ (a): Let A and B be the disjoint p*gb-closed sets in X. By (iv), there exist open sets U and V containing A and B respectively such that cl(U)∩cl(V)=ϕ. Since U∩V⊆cl(U) ∩ cl(V)=ϕ, U and V are disjoint open sets containing A and B respectively. Thus, X is strongly p*gb-normal.

Corollary 4.4. In a topological space X, the following are equivalent:

(a) X is strongly p*gb-normal.

(b) For every closed set A in X and every open set U containing A, there exists an open set V containing A such that cl(V)⊆U.

(c) For each pair of disjoint closed sets, A and B in X, there exists an open set U containing A such that cl(U)∩B=ϕ.

(d) For each pair of disjoint closed sets, A and B in X, there exist open sets U and V containing A and B respectively such that cl(U)∩cl(V)=ϕ.

Proof. Since every closed set is p*gb-closed and follows from the above theorem.

Theorem 4.5. A space (X,τ) is strongly p*gb-normal if and only if for every p*gb-closed set F and p*gb-open set G containing F, there exists an open set V such that F⊆V⊆cl(V)⊆G.

Proof: Let (X,τ) be strongly p*gb-normal. Let F be a p*gb-closed set and let G be a p*gb-open set containing F. Then F and X\G are disjoint p*gb-closed sets. Since X is strongly p*gb-normal, there exist disjoint open sets V_1 and V_2 such that F⊆V_1 and X\G⊆V_2. Thus F⊆V_1⊆X\$V_2$⊆G. Since X\$V_2$ is closed, so cl(V_1)⊆cl(X\$V_2$) =X\$V_2$⊆G. Take V=$V_1$. This implies ⊆V⊆cl(V)⊆G. Conversely suppose the condition holds. Let H_1 and H_2 be two disjoint p*gb-closed sets in X. Then X\$H_2$ is a p*gb-open set containing H_1. By assumption, there exists an open set V such that H_1⊆V⊆cl(V)⊆X\$H_2$. Since V is open and cl V) is closed. Then X\cl(V) is open. Now cl(V)⊆X\$H_2$ implies that H_2⊆X\cl(V). Also, V∩(X\cl(V)⊆cl(V)∩(X\cl(V)) =ϕ. That is V and X\cl(V) are disjoint open sets containing H_1 and H_2 respectively. This shows that (X,τ) is strongly p*gb-normal.

Theorem 4.6. For a space X, then the following are equivalent:

(a) X is strongly p*gb-normal.

(b) For any two p*gb-open sets U and V whose union is X, there exist closed subsets A of U and B of V whose union is also X.

Proof: (a)→(b): Let U and V be two p*gb-open sets in a strongly p*gb-normal space X such that X=U∪V. Then X\U, X\V are disjoint p*gb-closed sets. Since X is strongly p*gb-normal, then there exist disjoint open sets G_1 and G_2 such that X\U⊆G1 and X\V⊆G_2. Let A=X\$G_1$ and B=X\$G_2$. Then A and B are closed subsets of U and V respectively such that A∪B=X. This proves (b).

(b)→(a): Let A and B be disjoint p*gb-closed sets in X. Then X\A and X\B are p*gb-open sets whose union is X. By (b), there exist closed sets F_1 and F_2 such that F_1⊆X\A, F_2⊆X\B and F_1∪F_2=X. Then X\$F_1$ and X\$F_2$ are disjoint open sets containing A and B respectively. Therefore, X is strongly p*gb-normal.

Theorem 4.7. If given a pair of disjoint p*gb-closed sets A, B of X, there is a continuous function f:X → [0,1] such that f(A)={0} and f(B)={1}, then (X,τ) is strongly p*gb-normal.

Proof: Let (X,τ) be a topological space. Suppose for any pair of disjoint p*gb-closed sets A, B in X, there exists a continuous map f:X → [0,1] such that f(A)={0} and f(B)={1}. Let E and F be disjoint p*gb-closed sets in X. Let a, b∈[0,1] be arbitrary such that a≤b. Take G=[0, a) and H=(b,1]. Then and H are disjoint pen sets in [0, 1] Since f is continuous, f^{-1}(G) and f^{-1}(H) are open in X. By our assumption, f(E)= {0} and f(V)= {1}. Now f(E)= {0} implies f^{-1}(f(E))⊆ f^{-1}({0})⇒E⊆ f^{-1}(f(E))⊆ f^{-1}({0})⇒ E⊆ f^{-1}({0}). Similarly, F⊆f^{-1}({1}). Evidently, {0}⊆[0, a)⇒ f^{-1}({0})⊆f^{-1}([0, a)). This implies that E⊆f^{-1}({0})⊆f^{-1}([0, a)) = f^{-1}(G). Also {1}⊆(b, 1]⇒ f^{-1}({1})⊆ f^{-1}((b, 1]) which implies F⊆ f^{-1}({1})⊆ f^{-1}((b, 1])= f^{-1}(H). Further, f^{-1}(G)∩f^{-1}(H)=f^{-1}(GH)= f^{-1}(ϕ)=ϕ. So, we have a pair of disjoint open sets, f^{-1}(G), f^{-1}(H)⊆X such that ⊆f^{-1}(G) and F⊆f^{-1}(H). This proves that (X,τ) is strongly p*gb-normal.

Theorem 4.8. If f is a continuous and closed injection of a topological space X into a normal space Y and if every p*gb-closed set in X is closed, then X is p*gb-normal.

Proof: Let A and B be disjoint p*gb-closed sets in X. By assumption, A and B are closed in X. Then f(A) and f(B) are disjoint closed sets in Y. Since Y is normal, there exist disjoint open sets V_1 and V_2 in Y such that f(A)⊆V_1 and f(B)⊆V_2. Hence X is p*gb- normal.

Theorem 4.9. If f is a continuous and open bijection of a normal space X into a space Y and if every p*gb-closed set in Y is closed, then Y is p*gb-normal.

Proof: Let A and B be p*gb-closed set in Y. Then by assumption, B is closed in Y. Since f is a continuous function**,** f^{-1}(A) and f^{-1}(B) is a closed set in X. Since X is normal, there exist disjoint open sets U_1 and U_2 in X such that f^{-1}(A)⊆U_1 and f^{-1}(B)⊆U_2. Since f is open, f(U_1) and f (U_2) are disjoint open sets in Y containing A and B respectively. Hence Y is strongly p*gb-normal.

5. Weakly p*gb-normal spaces

Definition 5.1. A space X is said to be **weakly p*gb-normal** if, for every pair of disjoint closed sets, A and B in X, there are disjoint p*gb-open sets U and V in X containing A and B respectively.

Theorem 5.2. (i) Every normal space is weakly p*gb-normal.
(ii) Every p*gb-normal space is weakly p*gb-normal.

Proof: Suppose X is normal. Let A and B be disjoint closed sets in X. Since X is normal, there exist disjoint open sets U and V containing A and B respectively. Then U and V are p*gb-open in X. This implies that X is weakly p*gb-normal. This proves (i).

Suppose X is p*gb-normal. Let A and B be disjoint closed sets in X. Then A and B are disjoint p*gb-closed sets in X. Since X is p*gb-normal, there exist disjoint p*gb-open sets U and V containing A and B respectively. Therefore, X is weakly p*gb-normal. This proves (iii).

Theorem 5.3. In a topological space X, the following are equivalent:

(a) X is weakly p*gb-normal.

(b) For every closed set F in X and every open set U containing F, there exists a p*gb-open set V containing F such that p*gbcl(V)⊆U.

(c) For each pair of disjoint closed sets, A and B in X, there exists a p*gb-open set U containing A such that p*gbcl(U)∩B=ϕ.

Proof: (a)→(b): Let U be an open set containing the closed set F. Then H=X\U is a closed set disjoint from F. Since X is weakly p*gb-normal, there exist disjoint p*gb-open sets V and W containing F and H respectively. Then p*gbcl(V) is disjoint from H, since if y∈H, the set W is a p*gb-open set containing y disjoint from V. Hence p*gbcl(V)⊆U.

(b)→(c): Let A and B be disjoint closed sets in X. Then X\B is an open set containing A. By (b), there exists a p*gb-open set U containing A such that p*gbcl(U)⊆X\B. Hence p*gbcl(U)∩B=ϕ. This proves (c).

(c)→ (d): Let A and B be the disjoint p*gb-closed sets in X. By (iii), there exists a p*gb-open set U containing A such that p*gbcl(U)∩B=ϕ. Take V=X\p*gbcl(U). Then U and V are disjoint p*gb-open sets containing A and B respectively. Thus, X is weakly p*gb-normal.

Theorem 5.4. For a space X, then the following are equivalent:

(a) X is weakly p*gb-normal.

(b) For any two open sets U and V whose union is X, there exist p*gb-closed subsets A of U and B of V whose union is also X.

Proof: (a)→(b): Let U and V be two open sets in a weakly p*gb-normal space X such that X=U∪V. Then X\U, X\V are disjoint closed sets. Since X is weakly p*gb-normal, then there exist disjoint p*gb-open sets G_1 and G_2 such that X\U⊆G_1 and X\V⊆G_2. Let A=X\$G_1$ and B=X\$G_2$. Then A and B are p*gb-closed subsets of U and V respectively such that A∪B=X. This proves (b).

(b)→(a): Let A and B be disjoint closed sets in X. Then X\A and X\B are open sets whose union is X. By (ii), there exists p*gb-closed sets F_1 and F_2 such that F_1⊆X\A, F_2⊆X\B, and F_1∪F_2=X. Then X\$F_1$ and X\$F_2$ are disjoint p*gb-open sets containing A and B respectively. Therefore, X is weakly p*gb-normal.

Theorem 5.5. Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a function.

(a) If f is injective, continuous, p*gb-open and X is weakly p*gb-normal then Y is weakly p*gb-normal.
(b) If f is p*gb-irresolute, p*gb-closed and Y is weakly p*gb-normal then X is weakly p*gb-normal.

Proof: (a) Suppose X is weakly p*gb-normal. Let A and B be disjoint closed sets in Y. Since f is continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are closed in X. Since X is weakly p*gb-normal, there exist disjoint p*gb-open sets U and V in X such that $f^{-1}(A)\subseteq U$ and $f^{-1}(B)\subseteq V$.

Now $f^{-1}(A) \subseteq U \Rightarrow A\subseteq f(U)$ and $f^{-1}(B)\subseteq V\Rightarrow B\subseteq f(V)$. Since f is a p*gb-open map, f(U) and f(V) are p*gb-open in Y. Also, $U\cap V=\phi\Rightarrow f(U\cap V) =\phi$ and f is injective, then $f(U)\cap f(V)=\phi$. Thus f(U) and f(V) are disjoint p*gb-open sets in Y containing A and B respectively. Thus, Y is weakly p*gb-normal.

(b)Suppose Y is p*gb-normal. Let A and B be disjoint closed sets in X. Since f is p*gb-irresolute and p*gb-closed, f(A) and f(B) are p*gb-closed in Y. Since Y is p*gb-normal, there exist disjoint p*gb-open sets U and V in Y such that $f(A)\subseteq U$ and $f(B)\subseteq V$.

That is $A\subseteq f^{-1}(U)$ and $B\subseteq f^{-1}(V)$. Since f is p*gb-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint p*gb-open such that $A\subseteq f^{-1}(U)$ and $B\subseteq f^{-1}(V)$. Thus, X is p*gb-normal.

REFERENCES

[1] Andrijevic D, On b-open sets, Mat.Vesnik 48(1996), 59-64.

[2] Levine N, Generalized closed sets in topology, Rand. Circ. Mat. Palermo, 19(2)(1970), 89- 96.

[3] Selvi, T, Punitha Dharani, A, Some New Class of Nearly Closed and Open Sets, Asian Journal of Current Engineering and Maths, vol. 1(5), pp.305-307 (2012).

[4] Aruna Glory Sudha. I, Zion Chella Ruth S, More on p*gb-Closed Sets in Topological Spaces, International Journal of Mathematical Archive, 14(1), 2023, ISSN: 2229 5046,10-14.

[5] Willard S., General Topology, Addison Wesley, 1970.