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Connectedness And Compactness In Intuitionistic Topological Space

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ABSTRACT

In this paper, the notion of $\mathrm{I}\alpha_{\mathrm{g}}^{\wedge}$-connectedness in intuitionistic topological spaces. Also $\mathrm{I}\alpha_{\mathrm{g}}^{\wedge}$-compactness is defined in intuitionistic topological spaces and several preservation properties are obtained.

1 Introduction

The concept of intuitionistic sets in topological spaces was first introduced by Coker[1] in 1996. He also introduced the concept of intuitionistic points and investigated some fundamental properties of closed sets in intuitionistic topological spaces. Also in 2000 [3], developed the concept of intuitionistic topological spaces with intuitionistic sets and compactness. J.Arul Jesti and K.Heartlin [6] introduced the concept of alpha^generalized closed sets in intuitionistic topological spaces. In this paper some properties of $\mathrm{I}\alpha_{\mathrm{g}}^{\wedge}$- connectedness in intuitionistic topological spaces and $\mathrm{I}\alpha_{\mathrm{g}}^{\wedge}$-compactness in intuitionistic topological spaces.

2. Preliminaries

Definition 2.1 [1]: Let $\mathcal{H}$ be a non-empty set. An intuitionistic set (IS for short) $\mathcal{A}$ is an object having the form $\mathcal{A}$ $=<\mathcal{H}, \mathcal{A}_1, \mathcal{A}_2>$ Where $\mathcal{A}_1$ and $\mathcal{A}_2$ are subsets of $\mathcal{H}$ satisfying $\mathcal{A}_1 \cap \mathcal{A}_2 = \phi$. The set $\mathcal{A}_1$ is called the set of members of $\mathcal{A}$, while $\mathcal{A}_2$ is called set of non members of $\mathcal{A}$.

Definition 2.2 [1]: Let $\mathcal{H}$ be a non-empty set and $\mathcal{A}$ and $\mathfrak{B}$ are intuitionistic set in the form $\mathcal{A} = <\mathcal{H}, \mathcal{A}_1, \mathcal{A}_2>$, $\mathfrak{B} = <\mathcal{H}, \mathfrak{B}_1, \mathfrak{B}_2>$ respectively. Then

a) $\mathcal{A} \subseteq \mathfrak{B}$ iff $\mathcal{A}_1 \subseteq \mathfrak{B}_1$ and $\mathcal{A}_2 \supseteq \mathfrak{B}_2$
b) $\mathcal{A} = \mathfrak{B}$ iff $\mathcal{A} \subseteq \mathfrak{B}$ and $\mathfrak{B} \subseteq \mathcal{A}$
c) $\mathcal{A}^C = <\mathcal{H}, \mathcal{A}_2, \mathcal{A}_1>$
d) $\mathcal{A} - \mathfrak{B} = \mathcal{A} \cap \mathfrak{B}^C$
e) $\phi = <\mathcal{H}, \phi, \mathcal{H}>, \mathcal{H} = <\mathcal{H}, \mathcal{H}, \phi>$
f) $\mathcal{A} \cup \mathfrak{B} = <\mathcal{H}, \mathcal{A}_1 \cup \mathfrak{B}_1, \mathcal{A}_2 \cap \mathfrak{B}_2>$
g) $\mathcal{A} \cap \mathfrak{B} = <\mathcal{H}, \mathcal{A}_1 \cap \mathfrak{B}_1, \mathcal{A}_2 \cup \mathfrak{B}_2>$.

Definition 2.3 [5]: Let ($\mathcal{H}$,Iτ) be an intuitionistic topological space. Then $\mathcal{H}$ is called **I-disconnected** if there exists an I-open sets $R \neq \tilde{\phi}$ and $S \neq \tilde{\phi}$ such that $R \cup S \neq \tilde{\phi}$ and $R \cap S \neq \tilde{\phi}$. $\mathcal{H}$ is called **I-connected**, if $\mathcal{H}$ is not disconnected.

Definition 2.4 [5]: Let N be an intuitionistic set in the ITS ($\mathcal{H}$,Iτ). If there exists I-open sets R and S in $\mathcal{H}$ satisfying the following properties, then N is called IC_i-disconnected (i = 1,2,3,4).

IC_1: $N \subseteq R \cup S, R \cap S \subseteq N^c, N \cap R \neq \tilde{\phi}, N \cap S \neq \tilde{\phi}$,

IC_2: $N \subseteq R \cup S, R \cap S \cap N = \tilde{\phi}, N \cap R \neq \tilde{\phi}, N \cap S \neq \tilde{\phi}$.

IC_3: $N \subseteq R \cup S, R \cap S \subseteq N^c, R \nsubseteq N^c, S \nsubseteq N^c$,

IC_4: $N \subseteq R \cup S, R \cap S \cap N = \tilde{\phi}, R \nsubseteq N^c, S \nsubseteq N^c$.

N is said to be **IC_i-connected (i = 1,2,3,4)** if N is not IC_i-disconnected (i = 1,2,3,4).

Definition 2.5[7]: An ITS $\mathcal{H}$ is called $I\alpha_g^{\wedge}$-disconnected if there exists an $I\alpha_g^{\wedge}$-open sets $R \neq \phi$ and $S \neq \tilde{\phi}$ such that $R \cup S = \tilde{\mathcal{H}}$ and $R \cap S = \tilde{\phi}$. $\mathcal{H}$ is called $I\alpha_g^{\wedge}$-connected, if $\mathcal{H}$ is not $I\alpha_g^{\wedge}$-disconnected.

Definition 2.6 [7]: Let N be an intuitionistic set in the ITS ($\mathcal{H}$,Iτ). If there exists $I\alpha_g^{\wedge}$-open sets R and S in $\mathcal{H}$ satisfying the following properties, then N is called $I\alpha_g^{\wedge}C_i$-disconnected (i = 1,2,3,4).

$I\alpha_g^{\wedge}C_1$: $N \subseteq R \cup S, R \cap S \subseteq N^c, N \cap R \neq \tilde{\phi}, N \cap S \neq \tilde{\phi}$,

$I\alpha_g^{\wedge}C_2$: $N \subseteq R \cup S, R \cap S \cap N = \tilde{\phi}, N \cap R \neq \tilde{\phi}, N \cap S \neq \tilde{\phi}$.

$I\alpha_g^{\wedge}C_3$: $N \subseteq R \cup S, R \cap S \subseteq N^c, R \nsubseteq N^c, S \nsubseteq N^c$,

$I\alpha_g^{\wedge}C_4$: $N \subseteq R \cup S, R \cap S \cap N = \tilde{\phi}, R \nsubseteq N^c, S \nsubseteq N^c$.

N is said to be $I\alpha_g^{\wedge}$**C_i-connected (i = 1,2,3,4)** if N is not $I\alpha_g^{\wedge}C_i$-disconnected (i = 1,2,3,4).

Definition 2.7 [3]: Let $(\mathcal{H}, \mathrm{I\tau})$ be an intuitionistic topological space. If a family $\{< \mathcal{H}, \mathrm{P}_{1_k}, \mathrm{P}_{2_k} >; \mathrm{k} \in \mathrm{J}\}$ of I-open sets in $\mathcal{H}$ satisfies the condition $\cup \{< \mathcal{H}, \mathrm{P}_{1_k}, \mathrm{P}_{2_k} >; \mathrm{k} \in \mathrm{J}\} = \widetilde{\mathcal{H}}$, then it is called an **I-open cover** of $\mathcal{H}$. A finite subfamily of an $\mathrm{I}\alpha_g^{\Lambda}$-open cover $\{< \mathcal{H}, \mathrm{P}_{1_k}, \mathrm{P}_{2_k} >; \mathrm{k} \in \mathrm{J}\}$ of $\mathcal{H}$, which is also an I-open cover of $\mathcal{H}$ is called a finite I-subcover of $\{< \mathcal{H}, \mathrm{P}_{1_k}, \mathrm{P}_{2_k} >; \mathrm{k} \in \mathrm{J}\}$

Definition 2.8 [3]: Let $(\mathcal{H}, \mathrm{I\tau})$ be an intuitionistic topological space. A family $\{< \mathcal{H}, \mathrm{P}_{1_k}, \mathrm{P}_{2_k} >; \mathrm{k} \in \mathrm{J}\}$ of I-closed sets in $\mathcal{H}$ satisfies the finite intersection property iff every finite subfamily $\{\mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3, \ldots, \mathrm{P}_n\}$ of P satisfies the condition $\bigcap_{k=1}^{n} < \mathcal{H}, \mathrm{P}_{1_k}, \mathrm{P}_{2_k} > \neq \widetilde{\phi}$.

Definition 2.9 [3]: An ITS $(\mathcal{H}, \mathrm{I\tau})$ is said to be I-compact iff each I-open cover has a finite I-subcover

Definition 2.10 [3]: Let $(\mathcal{H}, \mathrm{I\tau})$ be an intuitionistic topological space and G be an IS in $\mathcal{H}$. The family $\{< \mathcal{H}, \mathrm{P}_{1_k}, \mathrm{P}_{2_k} >; \mathrm{k} \in \mathrm{J}\}$ of I-open sets in $\mathcal{H}$ is called an I-open cover of G if $\mathrm{G} \subseteq \cup \{< \mathcal{H}, \mathrm{P}_{1_k}, \mathrm{P}_{2_k} >; \mathrm{k} \in \mathrm{J}\}$.

Definition 2.11 [3]: An IS $\mathrm{G} = < \mathcal{H}, \mathrm{G}_1, \mathrm{G}_2 >$ in an ITS $(\mathcal{H}, \mathrm{I\tau})$ is called I-compact iff every I-open cover of G has a finite I-subcover. Also we can define an IS $\mathrm{G} = < \mathcal{H}, \mathrm{G}_1, \mathrm{G}_2 >$ in $(\mathcal{H}, \mathrm{I\tau})$ is I-compact iff for each family $\mathrm{P} = \{\mathrm{P}_k : \mathrm{k} \in \mathrm{J}\}$ where $\mathrm{P}_k = \{< \mathcal{H}, \mathrm{P}_{1_k}, \mathrm{P}_{2_k} >; \mathrm{k} \in \mathrm{J}\}$ of I-open sets in $\mathcal{H}$, $\mathrm{G}_1 \subseteq \bigcup_{k \in J} \mathrm{P}_{1_k}$ and $\mathrm{G}_2 \supseteq \bigcup_{k \in J} \mathrm{P}_{2_k}$, there exists a finite subfamily $\{\mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3, \ldots, \mathrm{P}_n\}$ of P such that $\mathrm{G}_1 \subseteq \bigcup_{k=1}^{n} \mathrm{P}_{1_k}$ and $\mathrm{G}_2 \supseteq \bigcup_{k=1}^{n} \mathrm{P}_{2_k}$

Proposition 2.12 [3]: Let $(\mathcal{H}, \mathrm{I\tau})$ be an intuitionistic topological space on $\mathcal{H}$. Then, we can also construct several ITS's on $\mathcal{H}$ in the following way:

(a) $\mathrm{I\tau}_{0,1}$={[ ] G : G ∈ Iτ}, (b) $\mathrm{I\tau}_{0,2}$={< > G : G ∈ Iτ}.

Remark 2.13 [3]: Let $(\mathcal{H}, \mathrm{I\tau})$ be an intuitionistic topological space.

(a) $\mathrm{I\tau}_1 = \{\mathcal{A}_1 : < \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2 > \in \mathrm{I\tau}\}$ is a topological space on $\mathcal{H}$. Similarly, $\mathrm{k}_2 = \{\mathcal{A}_2 : < \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2 > \in \mathrm{I\tau}\}$ is the family of all closed sets of the topological space $\mathrm{I\tau}_2 = \{(\mathcal{A}_2)^c : < \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2 > \in \mathrm{I\tau}\}$ on $\mathcal{H}$.

(b) Since $\mathcal{A}_1 \cap \mathcal{A}_2 = \phi$ for each $\mathcal{A} = < \mathcal{H}, \mathcal{A}_1, \mathcal{A}_2 > \in \mathrm{I\tau}$, we obtain $\mathcal{A}_1 \subseteq (\mathcal{A}_2)^c$ and $\mathcal{A}_2 \subseteq (\mathcal{A}_1)^c$. Hence, we may conclude that ($\mathcal{H}$, $I\tau_1$, $I\tau_2$) is a bitopological space.

3. $I\alpha_g^\wedge$-connectedness in Intuitionistic Topological Spaces

Theorem 3.1.: If G_x and G_y are intersecting $I\alpha_g^\wedge C_1$-connected sets, then $G_x \cup G_y$ is also $I\alpha_g^\wedge C_1$-connected.

Proof: Let $G_x \cup G_y$ be $I\alpha_g^\wedge C_1$-disconnected. Then there exists $I\alpha_g^\wedge$-open sets R and S such that $G_x \cup G_y \subseteq R \cup S$, $R \cap S \subseteq (G_x \cup G_y)^c$and $(G_1 \cup G_y) \cap R \neq \tilde{\phi}$, $(G_x \cup G_y) \cap S \neq \tilde{\phi}$. Suppose G_x and G_y are $I\alpha_g^\wedge C_1$-connected then ($G_x \cap R = \tilde{\phi}$ or $G_x \cap S = \tilde{\phi}$) and ($G_y \cap R = \tilde{\phi}$ or $G_y \cap S = \tilde{\phi}$). Since $G_x \cap G_y \neq \tilde{\phi}$, there exists $\tilde{\tilde{p}} \in G_x \cap G_y$ of the following cases. Case (i): Let $G_x \cap R = \tilde{\phi}$ and $G_y \cap S = \tilde{\phi}$. Then $(G_x \cap R) \cup (G_y \cap R) = (G_x \cup G_y) \cap R = \tilde{\phi}$ which is a contradiction. Case (ii): Let $G_x \cap R = \tilde{\phi}$ and $G_y \cap S = \tilde{\phi}$. Then there exists $\tilde{\tilde{p}} \notin R$, $\tilde{\tilde{p}} \notin S$ which is impossible since $\tilde{\tilde{p}} \in G_x \cup G_y \subseteq \mathcal{H} \cup \mathcal{Y}$. Case (iii): Let $G_x \cap S = \tilde{\phi}$ and $G_y \cap R = \tilde{\phi}$. Then there exists $\tilde{\tilde{p}} \notin R$, $\tilde{\tilde{p}} \notin S$ which is impossible as above. Case (iv): Let $G_x \cap S = \tilde{\phi}$ and $G_y \cap S = \tilde{\phi}$. Then $(G_x \cap S) \cup (G_y \cap S) = (G_x \cup G_y) \cap S = \tilde{\phi}$ which is a contradiction. Hence G_x and G_y are $I\alpha_g^\wedge C_1$-disconnected.

Theorem 3.2: If G_x and G_y are intersecting $I\alpha_g^\wedge C_2$-connected sets, then $G_x \cup G_y$ is also $I\alpha_g^\wedge C_2$-connected **.**

Proof: Similar to Theorem 3.1.

Theorem 3.3: Let (G_k): $k \in J$ be a family of $I\alpha_g^\wedge C_1$-connected sets such that $\cap\, G_k \neq \tilde{\phi}$. Then $\cup\, G_k$ is also $I\alpha_g^\wedge C_1$-connected.

Proof: Let $G = \cup\, G_k$ be $I\alpha_g^\wedge C_1$-disconnected. Then there exists $I\alpha_g^\wedge$-open sets R and S such that $G \subseteq R \cup S$, $R \cap S \subseteq G^c$, $G \cap R \neq \tilde{\phi}$, $G \cap S \neq \tilde{\phi}$. Consider any index $k_0 \in J$. Since G_{k_0} is $I\alpha_g^\wedge C_1$-connected, we have $G_{k_0} \cap R = \tilde{\phi}$, or $G_{k_0} \cap S = \tilde{\phi}$, So we have three cases. Case (i): If $G_k \cap R = \tilde{\phi}$ for each $k \in J_1$ and $G_k \cap R = (\cup\, G_k) \cap R = \cup\, (G_k \cap R) = \tilde{\phi}$ which is a contradiction. Case (ii): If $G_k \cap S = \tilde{\phi}$ for each $k \in J_1$ and $G_k \cap S = (\cup\, G_k) \cap S = \cup\, (G_k \cap S) = \tilde{\phi}$ which is a contradiction. Case (iii): If $G_k \cap R = \tilde{\phi}$ for each $k \in J_1$ and $G_k \cap S = \tilde{\phi}$ for each $k \in J_2$ where $J = J_1 \cup J_2$ and $J_1 \neq \tilde{\phi}$, $J_2 \neq \tilde{\phi}$. Since $\cap\, G_k \neq \tilde{\phi}$, $\tilde{\tilde{p}} \in \cap\, G_k$. In this case $\tilde{\tilde{p}} \notin R$ and $\tilde{\tilde{p}} \notin S$ which is a contradiction $\tilde{\tilde{p}} \in G \subseteq R \cup S$. Hence G is also $I\alpha_g^\wedge C_1$-disconnected.

Theorem 3.4: Let $(G_k): k \in J$ be a family of $I\alpha_g^{\wedge}C_2$-connected sets such that $\cap\, G_k \neq \tilde{\phi}$. Then $\cup\, G_k$ is also $I\alpha_g^{\wedge}C_2$-connected.

Proof : Similar to Theorem 3.3.

Theorem 3.5: Let $(\mathcal{H}, I\tau)$ be an intuitionistic topological space. Then
(1) $\tilde{a}$ is $I\alpha_g^{\wedge}C_1$-connected
(2) $\tilde{a}$ is $I\alpha_g^{\wedge}C_2$-connected.

Proof. (1) Suppose $\tilde{a}$ be $I\alpha_g^{\wedge}C_1$-disconnected. Then there exist $I\alpha_g^{\wedge}$open sets R and S such that $\tilde{a} \subseteq R \cup S$, $R \cap S \subseteq \tilde{a}^c$, $\tilde{a} \cap R \neq \tilde{\phi}$, $\tilde{a} \cap S \neq \tilde{\phi}$ where $\tilde{a}^c = < \mathcal{H}, \{a\}^c, \{a\} >$. Since $\tilde{a} \cap R \neq \tilde{\phi}$ and $\tilde{a} \cap S \neq \tilde{\phi}$, we get $\tilde{\tilde{a}} \in R$ and $\tilde{\tilde{a}} \in S$. But $R \cap S \subseteq \tilde{a}^c$ implies $R_1 \cap S_1 \subseteq \tilde{a}^c$ and $R_2 \cup S_2 \supseteq \tilde{a}^c$ which is impossible. Hence $\tilde{a}$ is $I\alpha_g^{\wedge}C_1$-connected. (ii) Let $\tilde{a}$ be $I\alpha_g^{\wedge}C_2$-disconnected. Then there exist $I\alpha_g^{\wedge}$-open sets R and S such that $\tilde{a} \subseteq R \cup S$, $R \cap S \cap \tilde{a} = \tilde{\phi}$, $\tilde{a} \cap R \neq \tilde{\phi}$, $\tilde{a} \cap S \neq \tilde{\phi}$. Since $\tilde{a} \cap R \neq \tilde{\phi}$ and $\tilde{a} \cap S \neq \tilde{\phi}$, we get $\tilde{\tilde{a}} \in R$ and $\tilde{\tilde{a}} \in S$ which implies $a \notin R_2$ and $a \notin S_2$. But $R \cap S \cap \tilde{a} = \tilde{\phi}$ which implies $R_2 \cup S_2 \cup \{a\}^c = \tilde{\mathcal{H}}$ which is impossible. Hence $\tilde{a}$ is $I\alpha_g^{\wedge}C_2$-connected.

4. $I\alpha_g^{\wedge}$-Compactness In Intuitionistic Topological Spaces

In this section we discuss the concepts of an $I\alpha_g^{\wedge}$-Compactness in intuitionistic topological spaces and also some results are discussed.

Definition 4.1: Let $(\mathcal{H}, I\tau)$ be an intuitionistic topological space. If a family $\{< \mathcal{H}, P_{1_k}, P_{2_k} >; k \in J\}$ of $I\alpha_g^{\wedge}$-open sets in $\mathcal{H}$ satisfies the condition $\cup\, \{< \mathcal{H}, P_{1_k}, P_{2_k} >; k \in J\} = \tilde{\mathcal{H}}$, then it is called an $I\alpha_g^{\wedge}$-open cover of $\mathcal{H}$. A finite subfamily of an $I\alpha_g^{\wedge}$-open cover $\{< \mathcal{H}, P_{1_k}, P_{2_k} >; k \in J\}$ of $\mathcal{H}$, which is also an $I\alpha_g^{\wedge}$-open cover of $\mathcal{H}$ is called a finite $I\alpha_g^{\wedge}$-subcover of $\{< \mathcal{H}, P_{1_k}, P_{2_k} >; k \in J\}$

Definition 4.2: Let $(\mathcal{H}, I\tau)$ be an intuitionistic topological space. A family $\{< \mathcal{H}, P_{1_k}, P_{2_k} >; k \in J\}$ of $I\alpha_g^{\wedge}$-closed sets in $\mathcal{H}$ satisfies the finite intersection property iff every finite subfamily $\{P_1, P_2, P_3,\ldots, P_n\}$ of P satisfies the condition $\cap_{k=1}^{n} < \mathcal{H}, P_{1_k}, P_{2_k} > \neq \tilde{\phi}$.

Definition 4.3: An ITS $(\mathcal{H}, I\tau)$ is said to be $I\alpha_g^{\wedge}$-compact iff each $I\alpha_g^{\wedge}$-open cover has a finite $I\alpha_g^{\wedge}$-subcover

Definition 4.4: Let $(\mathcal{H}, I\tau)$ be an intuitionistic topological space and G be an IS in $\mathcal{H}$. The family $\{< \mathcal{H}, P_{1_k}, P_{2_k} >; k \in J\}$ of $I\alpha_g^{\wedge}$-open sets in $\mathcal{H}$ is called an $I\alpha_g^{\wedge}$-open cover of G if $G \subseteq \cup \{< \mathcal{H}, P_{1_k}, P_{2_k} >; k \in J\}$.

Definition 4.5: An IS $G = < \mathcal{H}, G_1, G_2 >$ in an ITS $(\mathcal{H}, I\tau)$ is called $I\alpha_g^{\wedge}$-compact iff every $I\alpha_g^{\wedge}$-open cover of G has a finite $I\alpha_g^{\wedge}$-subcover. Also we can define an IS $G = < \mathcal{H}, G_1, G_2 >$ in $(\mathcal{H}, I\tau)$ is $I\alpha_g^{\wedge}$-compact iff for each family $P = \{P_k : k \in J\}$ where $P_k = \{< \mathcal{H}, P_{1_k}, P_{2_k} >; k \in J\}$ of $I\alpha_g^{\wedge}$-open sets in $\mathcal{H}$, $G_1 \subseteq \bigcup_{k \in J} P_{1_k}$ and $G_2 \supseteq \bigcup_{k \in J} P_{2_k}$, there exists a finite subfamily $\{P_1, P_2, P_3, \ldots, P_n\}$ of P such that $G_1 \subseteq \bigcup_{k=1}^{n} P_{1_k}$ and $G_2 \supseteq \bigcup_{k=1}^{n} P_{2_k}$.

Proposition 4.6: Let $(\mathcal{H}, I\tau)$ be an intuitionistic topological space. Then $(\mathcal{H}, I\tau)$ is $I\alpha_g^{\wedge}$-compact iff the ITS $(\mathcal{H}, I\tau_{0,1})$ is $I\alpha_g^{\wedge}$-compact.

Proof: Necessity: Let $(\mathcal{H}, I\tau)$ be $I\alpha_g^{\wedge}$-compact and consider an $I\alpha_g^{\wedge}$-open cover $\{[\,]P_k : k \in J\}$ of $\mathcal{H}$ in $(\mathcal{H}, I\tau_{0,1})$. Since $\cup ([\,]P_k) = \widetilde{\mathcal{H}}$, we obtain $\cup P_{1_k} = \mathcal{H}$ and hence $P_{2_k} \subseteq (P_{1_k})^c$ which implies $\cap P_{2_k} \subseteq (\cup P_{1_k})^c = \phi$ which implies $\cap P_{2_k} = \phi$ and hence $\cup P_k = \widetilde{\mathcal{H}}$. Since $(\mathcal{H}, I\tau)$ is $I\alpha_g^{\wedge}$-compact, there exists $P_1, P_2, P_3, \ldots, P_n$ such that $\bigcup_{k=1}^{n} P_k = \widetilde{\mathcal{H}}$ which implies $\bigcup_{k=1}^{n} P_{1_k} = \mathcal{H}$ and $\bigcap_{k=1}^{n} P_{2_k} = \phi$. Hence $(\mathcal{H}, I\tau_{0,1})$ is $I\alpha_g^{\wedge}$-compact.

Sufficiency: Suppose $(\mathcal{H}, I\tau_{0,1})$ is $I\alpha_g^{\wedge}$-compact. Consider an $I\alpha_g^{\wedge}$-open cover $\{P_k : k \in J\}$ of $\mathcal{H}$ in $(\mathcal{H}, I\tau)$. Since $\cup P_k = \widetilde{\mathcal{H}}$, we obtain $\cup P_{1_k} = \mathcal{H}$ and hence $\cap (P_{1_k})^c = \phi$ which implies $\cup ([\,]P_k) = \widetilde{\mathcal{H}}$. Since $(\mathcal{H}, I\tau_{0,1})$ is $I\alpha_g^{\wedge}$-compact, there exists $P_1, P_2, P_3, \ldots, P_n$ such that $\bigcup_{k=1}^{n}([\,]P_k) = \widetilde{\mathcal{H}}$ which implies $\bigcup_{k=1}^{n} P_{1_k} = \mathcal{H}$ and $\bigcap_{k=1}^{n}(P_{1_k})^c = \phi$. Hence $P_{1_k} \subseteq (P_{2_k})^c$ which implies $\mathcal{H} = \bigcup_{k=1}^{n} P_{1_k} \subseteq (\bigcap_{k=1}^{n} P_{2_k})^c$ which implies $\bigcap_{k=1}^{n} P_{2_k} = \phi$. Thus $\bigcup_{k=1}^{n} P_k = \widetilde{\mathcal{H}}$. So $(\mathcal{H}, I\tau)$ is $I\alpha_g^{\wedge}$-compact.

Proposition 4.7: The ITS $(\mathcal{H}, I\tau)$ is $I\alpha_g^{\wedge}$-compact iff $(\mathcal{H}, I\tau_\mu)$ is $I\alpha_g^{\wedge}$-compact.

Proof: Similar to Proposition 4.6.

Proposition 5.5.8: Let $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathcal{Y}, I\tau_{\theta})$ be a surjective $I\alpha_g^{\wedge}$-continuous mapping. If $(\mathcal{H}, I\tau_{\mu})$ is $I\alpha_g^{\wedge}$-compact then $(\mathcal{Y}, I\tau_{\theta})$ is intuitionistic compact.

Proof: Let $\{P_k : k \in J\}$ be any intuitionistic open cover of $\mathcal{Y}$. Since f is $I\alpha_g^{\wedge}$-continuous, $\{f^{-1}(P_k) : k \in J\}$ is an $I\alpha_g^{\wedge}$-open cover of $\mathcal{H}$. Since $(\mathcal{H}, I\tau_{\mu})$ is $I\alpha_g^{\wedge}$-compact, it has a finite subcover $\{f^{-1}(P_1), f^{-1}(P_2), f^{-1}(P_3),...., f^{-1}(P_n)\}$ such that $\bigcup_{k=1}^{n} f^{-1}(P_{1_k}) = \widetilde{\mathcal{H}}$ and $\bigcap_{k=1}^{n} f^{-1}(P_{2_k}) = \tilde{\phi}$ that is $f^{-1}(\bigcup_{k=1}^{n}(P_{1_k})) = \mathcal{H}$ and $f^{-1}(\bigcap_{k=1}^{n}(P_{2_k})) = \phi$ which implies $\bigcup_{k=1}^{n}(P_{1_k}) = f(\mathcal{H})$ and $\bigcap_{k=1}^{n}(P_{2_k}) = f(\phi)$. Since f is surjective $\{P_1, P_2, P_3,...., P_n\}$ is an open cover of $\mathcal{Y}$ and hence $(\mathcal{Y}, I\tau_{\theta})$ is intuitionistic compact.

Corollary 4.9: Let $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathcal{Y}, I\tau_{\theta})$ be $I\alpha_g^{\wedge}$-continuous. If N is $I\alpha_g^{\wedge}$-compact in $(\mathcal{H}, I\tau_{\mu})$, then $f(N)$ is intuitionistic compact in $(\mathcal{Y}, I\tau_{\theta})$.

Proof: Let $\{G_k : k \in J\}$ be an intuitionistic open set of $\mathcal{Y}$ such that $f(N) \subseteq \cup \{G_k : k \in J\}$. Then $N \subset \cup \{f^{-1}(G_k) : k \in J\}$ where $f^{-1}(G_k)$ is $I\alpha_g^{\wedge}$-open in $\mathcal{H}$ for each k. Since N is $I\alpha_g^{\wedge}$-compact relative to $\mathcal{H}$, there exists a finite sub collection $\{G_1, G_2, \ldots\ldots, G_n\}$ such that $N \subset \cup \{f^{-1}(G_k) : k = 1, 2, \ldots, n\}$. Hence $f(N) \subset f(\cup \{f^{-1}(G_k) : k = 1, 2, \ldots, n\}) = \cup \{f(f^{-1}(G_k)) : k = 1, 2, \ldots, n\}) \subset \cup \{G_k : k = 1, 2, \ldots, n\}$. Hence $f(N)$ is $I\alpha_g^{\wedge}$-compact relative to $\mathcal{Y}$.

Proposition 4.10: Let $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathcal{Y}, I\tau_{\theta})$ be an $I\alpha_g^{\wedge}$-irresolute mapping and if M is $I\alpha_g^{\wedge}$-compact relative to $\mathcal{H}$, then $f(M)$ is $I\alpha_g^{\wedge}$-compact relative to $\mathcal{Y}$.

Proof. Let $\{P_k : k \in J\}$ be an $I\alpha_g^{\wedge}$-open set of $\mathcal{Y}$ such that $f(M) \subseteq \cup \{P_k : k \in J\}$. Then $M \subset \cup \{f^{-1}(P_k) : k \in J\}$ where $f^{-1}(P_k)$ is $I\alpha_g^{\wedge}$-open in $\mathcal{H}$ for each k. Since M is $I\alpha_g^{\wedge}$-compact relative to $\mathcal{H}$, there exists a finite sub collection $\{P_1, P_2, \ldots\ldots, P_n\}$ such that $M \subset \cup \{f^{-1}(P_k) : k = 1, 2, \ldots, n\}$ which implies $f(M) \subset \cup \{P_k : k = 1, 2, \ldots, n\}$. Hence $f(M)$ is $I\alpha_g^{\wedge}$-compact relative to $\mathcal{Y}$.

Proposition 4.11: Let $f:(\mathcal{H},I\tau_{\mu}) \longrightarrow (\mathcal{Y},I\tau_{\theta})$ be an $I\alpha_g^{\wedge}$-irresolute mapping. If $\mathcal{H}$ is $I\alpha_g^{\wedge}$-compact, then $\mathcal{Y}$ is also an $I\alpha_g^{\wedge}$-compact space .

Proof. Let $f:(\mathcal{H},I\tau_{\mu}) \longrightarrow (\mathcal{Y},I\tau_{\theta})$ be an $I\alpha_g^{\wedge}$-irresolute mapping from $I\alpha_g^{\wedge}$-compact space $(\mathcal{H},I\tau_{\mu})$ onto an intuitionistic topological space $(\mathcal{Y},I\tau_{\theta})$. Let $\{G_k : k \in J\}$ be an $I\alpha_g^{\wedge}$-open cover of $\mathcal{Y}$. Then $\{f^{-1}(G_k) : k \in J\}$ is an $I\alpha_g^{\wedge}$-open cover of $\mathcal{H}$. Since $\mathcal{H}$ is $I\alpha_g^{\wedge}$-compact, there is a finite subfamily $\{f^{-1}(G_{k_1}), f^{-1}(G_{k_2}), f^{-1}(G_{k_3}), \ldots\ldots, f^{-1}(G_{k_n})\}$ of $\{f^{-1}(G_k) : k \in J\}$ such that $\bigcup_{j=1}^{n} f^{-1}(G_{k_j}) = \widetilde{\mathcal{H}}$. Since f is onto, $f(\widetilde{\mathcal{H}}) = \widetilde{\mathcal{Y}}$ and $f(\bigcup_{j=1}^{n} f^{-1}(G_{k_j})) = \bigcup_{j=1}^{n} f(f^{-1}(G_{k_j})) = \bigcup_{j=1}^{n} G_{k_j}$. It follows that $\bigcup_{j=1}^{n} G_{k_j} = \widetilde{\mathcal{Y}}$ and the family $\{G_{k_1}, G_{k_2}, G_{k_3}, \ldots\ldots, G_{k_n}\}$ is an intuitionistic finite subcover of $\{G_k : k \in J\}$. Hence $(\mathcal{Y}, I\tau_{\theta})$ is an $I\alpha_g^{\wedge}$- compact.

Theorem 4.12: An ITS $(\mathcal{H}, I\tau)$ is $I\alpha_g^{\wedge}$-compact iff every family $\{< \mathcal{H}, P_{1_k}, P_{2_k} > ; k \in J\}$ of $I\alpha_g^{\wedge}$-closed sets in A having the FIP has a nonempty intersection.

Proof: Assume that $\mathcal{H}$ is $I\alpha_g^{\wedge}$-compact that is every $I\alpha_g^{\wedge}$-open cover of $\mathcal{H}$ has a finite $I\alpha_g^{\wedge}$-subcover. Let $P_k = \{< \mathcal{H}, P_{1_k}, P_{2_k} > ; k \in J\}$ be a family of $I\alpha_g^{\wedge}$-closed sets of $\mathcal{H}$. Also assume that this family has finite intersection property. We have to show that $\bigcap_{k \in J} P_k \neq \widetilde{\phi}$. Suppose on the contrary , $\bigcap_{k \in J} P_k = \widetilde{\phi}$ which implies $\overline{\bigcap_{k \in J} P_k} = \overline{\widetilde{\phi}}$ which implies $\bigcup_{k \in J} \overline{P_k} = \widetilde{\mathcal{H}}$ that is $\bigcup_{k \in J} < \mathcal{H}, P_{2_k}, P_{1_k} > = \widetilde{\mathcal{H}}$. Since for every $k \in J$, P_k is an $I\alpha_g^{\wedge}$-closed set of $\mathcal{H}$, therefore $\overline{P_k}$ will be an $I\alpha_g^{\wedge}$-open set of $\mathcal{H}$. Thus, $\{ \overline{P_k} = < \mathcal{H}, P_{2_k}, P_{1_k} > : k \in J\}$ is an $I\alpha_g^{\wedge}$-open cover for $\mathcal{H}$. Since $\mathcal{H}$ is $I\alpha_g^{\wedge}$-compact, this $I\alpha_g^{\wedge}$-cover has a finite $I\alpha_g^{\wedge}$-subcover, say, $\bigcup_{k=1}^{n} \overline{P_k} = \bigcup_{k=1}^{n} < \mathcal{H}, P_{2_k}, P_{1_k} > = \widetilde{\mathcal{H}}$ .Then, $\overline{\bigcup_{k=1}^{n} \overline{P_k}} = \overline{\widetilde{\mathcal{H}}}$ which implies $\bigcap_{k=1}^{n} P_k = \widetilde{\phi}$ Thus, the above considered family does not satisfy the FIP which is a contradiction. Therefore, $\bigcap_{k \in J} P_k \neq \widetilde{\phi}$. Conversely, assume that the family of $I\alpha_g^{\wedge}$-closed sets of $\mathcal{H}$ having FIP has nonempty intersection. To show that $\mathcal{H}$ is $I\alpha_g^{\wedge}$-compact. Let $\{P_k = \{< \mathcal{H}, P_{1_k}, P_{2_k} > ; k \in J\}$ be an $I\alpha_g^{\wedge}$-open cover of $\mathcal{H}$. Suppose this $I\alpha_g^{\wedge}$-open cover has no finite $I\alpha_g^{\wedge}$-subcover, that is for every finite subcollection of the given cover, say, $\bigcup_{k=1}^{n} P_k \neq \widetilde{\mathcal{H}}$ which implies $\overline{\bigcup_{k=1}^{n} P_k} \neq \overline{\widetilde{\mathcal{H}}}$ which implies $\bigcap_{k=1}^{n} \overline{P_k} \neq \widetilde{\phi}$. As each P_k is an $I\alpha_g^{\wedge}$-open set of $\mathcal{H}$

therefore, each $\overline{\mathrm{P_k}}$ is an $\mathrm{I}\alpha_{\mathrm{g}}^{\wedge}$-closed set of $\mathcal{H}$. Thus, $\{\overline{\mathrm{P_k}} = <\mathcal{H}, \mathrm{P_{2_k}}, \mathrm{P_{1_k}} > : \mathrm{k} \in \mathrm{J}\}$ is a family of $\mathrm{I}\alpha_{\mathrm{g}}^{\wedge}$-closed set of $\mathcal{H}$ having FIP. Hence by the hypothesis it has nonempty intersection, that is $\bigcap_{\mathrm{k}\in\mathrm{J}} \overline{\mathrm{P_k}} \neq \widetilde{\phi}$ which implies $\overline{\bigcap_{\mathrm{k}\in\mathrm{J}} \overline{\mathrm{P_k}}} \neq \overline{\widetilde{\phi}}$ which implies $\bigcup_{\mathrm{k}\in\mathrm{J}} \mathrm{P_k} \neq \widetilde{\mathcal{H}}$. This shows that the family $\{\mathrm{P_k} = \{<\mathcal{H}, \mathrm{P_{1_k}}, \mathrm{P_{2_k}} > ; \mathrm{k} \in \mathrm{J}\}$ is not an $\mathrm{I}\alpha_{\mathrm{g}}^{\wedge}$-open cover for $\mathcal{H}$, which is a contradiction. Therefore, the given family must have a finite $\mathrm{I}\alpha_{\mathrm{g}}^{\wedge}$-subcover and hence $\mathcal{H}$ is $\mathrm{I}\alpha_{\mathrm{g}}^{\wedge}$-compact.

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