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$A(Gg)^*$- Locally Closed Sets In Topological Spaces

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Abstract

In this paper we introduce the concept of $\alpha(gg)^*$-lc , $\alpha(gg)^*$-lc* and $\alpha(gg)^*$-lc**sets using the concept of $\alpha(gg)^*$-open and $\alpha(gg)^*$-closed sets. Also, we present some of the weaker forms of locally continuous functions.

Keywords: $\alpha(gg)^*$-lc sets, $\alpha(gg)^*$-lc*sets, $\alpha(gg)^*$-lc**sets, $\alpha(gg)^*$-lc continuous functions

I. INTRODUCTION

In 1921, Kuratowski and Sierpinski [1] introduced the notion of locally closed sets in topological spaces. In 1989, Ganster and Reilly [5] introduced the concept of LC- continuous and LC-irresolute maps using locally closed sets. Later on, many researchers have investigated on this topic and introduced different types of locally closed sets. In 2022, authors [2] introduced $\alpha(gg)^*$- closed sets in topological spaces. In this paper, authors put forth the concept of $\alpha(gg)^*$-lc, $\alpha(gg)^*$-lc*and $\alpha(gg)^*$-lc** sets using the concept of $\alpha(gg)^*$- open and $\alpha(gg)^*$- closed sets. Later on, we discussed about $\alpha(gg)^*$ -LC continuous (resp. $\alpha(gg)^*$-lc irresolute) functions and some of its basic properties are examined

II. PRELIMINARIES

Throughout this paper (X, τ) represents the topological spaces on which no separation axioms are assumed unless otherwise mentioned. A being a subset of a topological space (X, τ), cl(A), int(A) denote the closure of A and interior of A respectively.

Definition.2.1.[2] A set A of a topological space (X, τ) is called alpha generalization of generalized star closed (briefly $\alpha(gg)^*$-

closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gg)^*$- open in (X, τ).

Definition.2.2.[5] A set A of a topological space (X, τ) is called locally closed if $A = G \cap F$, where G is open and F is closed.

Definition.2.4. A function $f: X \rightarrow Y$ is called

(i) $\alpha(gg)^*$-continuous [3] if $f^{-1}(V)$ is $\alpha(gg)^*$-closed in X for every closed subset V of Y.

(ii) $\alpha(gg)^*$-irresolute if $f^{-1}(V)$ is $\alpha(gg)^*$-closed in (X, τ) for every $\alpha(gg)^*$-closed subset V of (Y, σ).

Definition.2.5.[4] A topological space (X, τ) is said to be a door space iff every subset of X is either open or closed.

Lemma.2.6.[2] Every closed set is $\alpha(gg)^*$-closed.

III. MAIN RESULTS

Definition.3.1. A set A of a topological space (X, τ) is called $\alpha(gg)^*$-locally closed (briefly $\alpha(gg)^*$-lc) if $A = G \cap F$, where G is an $\alpha(gg)^*$-open set and F is an $\alpha(gg)^*$-closed set.

Theorem 3.2.

(i) Every $\alpha(gg)^*$-open set in X is $\alpha(gg)^*$-lc.

(ii) Every $\alpha(gg)^*$-closed set in X is $\alpha(gg)^*$-lc.

Proof.

(i) Let A be an $\alpha(gg)^*$-open set in X. Then $A = A \cap X$, where A is $\alpha(gg)^*$-open and X is $\alpha(gg)^*$-closed. Thus A is $\alpha(gg)^*$-lc.

(ii) Let A be an $\alpha(gg)^*$-closed set in X. Then $A = X \cap A$, where X is $\alpha(gg)^*$-open and A is $\alpha(gg)^*$-closed. Thus A is $\alpha(gg)^*$-lc.

Remark 3.3. The converse of the above theorem need not be true in general as seen fromthe following example.

Example 3.4. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

(i) $\{c, d\} = X \cap \{c, d\}$. Here $\{c, d\}$ is $\alpha(gg)^*$-lc but not $\alpha(gg)^*$-open.

(ii) $\{a\} = \{a, c\} \cap \{a, d\}$. Here $\{a\}$ is $\alpha(gg)^*$-lc but not $\alpha(gg)^*$-closed.

Theorem 3.5. In a topological space (X, τ)

Every open set is $\alpha(gg)^*$-lc.

(i) Every closed set is $\alpha(gg)^*$-lc.

Proof.

(i) Let $\mathbf{A}$ be an open set in $\mathbf{X}$. Then $\mathbf{A}$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-open in $\mathbf{X}$. Thus, $\mathbf{A}$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-lc in $\mathbf{X}$.

(ii) Let $\mathbf{A}$ be a closed set in $\mathbf{X}$. Then $\mathbf{A}$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-closed in $\mathbf{X}$. Thus, $\mathbf{A}$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-lc in $\mathbf{X}$.

Remark 3.6. The converse of the above theorem need not be true in general as seen from the following example.

Example 3.7. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

(i) $\{b, c\} = \{a, b, c\} \cap \{b, c, d\}$. Here $\{b, c\}$ is $\alpha(gg)^*$-lc but not open.

(ii) $\{a, b\} = \{a, b, c\} \cap \{a, b, d\}$. Here $\{a, b\}$is $\alpha(gg)^*$-lc but not closed.

Theorem 3.8. Every locally closed set is $\alpha(gg)^*$-locally closed.

Proof. Let A be a locally closed subset of X. Then $A = G \cap F$, where G is open and F is closed. That is, $A = G \cap F$, where G is $\alpha(gg)^*$-open and F is $\alpha(gg)^*$-closed. Thus, A is $\alpha(gg)^*$- locally closed in X.

Remark 3.9. The converse of the above theorem need not be true in general as seen fromthe following example.

Example 3.10. Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, \{a\}, \{b, c\}\}$.Then $\{c\} = \{a, c\} \cap \{b, c\}$is $\alpha(gg)^*$-locally closed but not locally closed.

Theorem 3.11. A subset A of X is $\alpha(gg)^*$-lc if and only if its complement, A^c is the union of an $\alpha(gg)^*$-open set and an $\alpha(gg)^*$-closed set.

Proof. Let A be an $\alpha(gg)^*$-lc set in (X, τ). Then $A = G \cap F$, where G is $\alpha(gg)^*$-openand F is $\alpha(gg)^*$-closed.This implies that, $A^c = (G \cap F)^c = F^c \cup G^c$, where F^c is $\alpha(gg)^*$-open and G^c is $\alpha(gg)^*$-closed. Conversely, assume that A^c is the union of an $\alpha(gg)^*$-open set and an $\alpha(gg)^*$-closed set. That is, $A^c = G \cup F$, where G is $\alpha(gg)^*$-open and F is $\alpha(gg)^*$-closed. This implies that $(A^c)^c = (G \cup F)^c$. That is, $A = F^c \cap G^c$, where F^c is $\alpha(gg)^*$-open and G^c is $\alpha(gg)^*$-closed. Thus, A is $\alpha(gg)^*$-lc set in X.

Theorem 3.12. Let $f: X \to Y$ be a continuous function. If K is locally closed subset of Y, then $f^{-1}(K)$ is $\alpha(gg)^*$-lc in X.

Proof. Let K be a locally closed subset of Y. Then $\mathrm{K} = \mathrm{G} \cap \mathrm{F}$, where G is open and F is closed. This implies that $\mathrm{f}^{-1}(\mathrm{K}) = \mathrm{f}^{-1}(\mathrm{G} \cap \mathrm{F}) = \mathrm{f}^{-1}(\mathrm{G}) \cap \mathrm{f}^{-1}(\mathrm{F})$, where $\mathrm{f}^{-1}(\mathrm{G})$ is openand $\mathrm{f}^{-1}(\mathrm{F})$ is closed. That is, $\mathrm{f}^{-1}(\mathrm{K}) = \mathrm{f}^{-1}(\mathrm{G}) \cap \mathrm{f}^{-1}(\mathrm{F})$, where $\mathrm{f}^{-1}(\mathrm{G})$ is $\alpha(\mathrm{gg})^*$-open and $\mathrm{f}^{-1}(\mathrm{F})$ is $\alpha(\mathrm{gg})^*$-closed. Thus, $\mathrm{f}^{-1}(\mathrm{K})$ is $\alpha(\mathrm{gg})^*$-lc in X.

Theorem 3.13. For a subset A of a topological space X

(i) A is $\alpha(\mathrm{gg})^*$-lc.
(ii) $\mathrm{A} = \mathrm{U} \cap \alpha(\mathrm{gg})^*\mathrm{cl}(\mathrm{A})$ for some $\alpha(\mathrm{gg})^*$-open set U.

Proof.

(i)⇒(ii)

Suppose A is $\alpha(\mathrm{gg})^*$-lc. Then $\mathrm{A} = \mathrm{U} \cap \mathrm{F}$, where U is $\alpha(\mathrm{gg})^*$-open and F is $\alpha(\mathrm{gg})^*$-closed. That is, $\alpha(\mathrm{gg})^*\mathrm{cl}(\mathrm{A}) = \alpha(\mathrm{gg})^*\mathrm{cl}(\mathrm{U} \cap \mathrm{F}) \subseteq \alpha(\mathrm{gg})^*\mathrm{cl}(\mathrm{U}) \cap \alpha(\mathrm{gg})^*\mathrm{cl}(\mathrm{F}) \subseteq \alpha(\mathrm{gg})^*\mathrm{cl}(\mathrm{F}) = \mathrm{F}$. Thus, $\alpha(\mathrm{gg})^*\mathrm{cl}(\mathrm{A}) \subseteq \mathrm{F}$. Then $\mathrm{A} \subseteq \mathrm{U} \cap \alpha(\mathrm{gg})^*\mathrm{cl}(\mathrm{A}) \subseteq \mathrm{U} \cap \mathrm{F} = \mathrm{A}$. Hence $\mathrm{A} = \mathrm{U} \cap \alpha(\mathrm{gg})^*\mathrm{cl}(\mathrm{A})$.

(ii)⇒(i)

Suppose that $\mathrm{A} = \mathrm{U} \cap \alpha(\mathrm{gg})^*\mathrm{cl}(\mathrm{A})$ for some $\alpha(\mathrm{gg})^*$-open set U. Since $\mathrm{cl}(\mathrm{A})$ is closed, by Lemma.2.6 the theorem follows.

IV. $\alpha(\mathrm{gg})^*$-locally closed* sets

Definition 4.1. A subset A of a topological space (X, τ) is an $\alpha(\mathrm{gg})^*$-locally closed* (briefly $\alpha(\mathrm{gg})^*$-lc*) set if there exists an $\alpha(\mathrm{gg})^*$-open set G and a closed set F in X such that $\mathrm{A} = \mathrm{G} \cap \mathrm{F}$. The set of all $\alpha(\mathrm{gg})^*\text{-lc}^*$ subsets of (X, τ) is denoted by $\alpha(\mathrm{GG})^*\text{-LC}^*(\mathrm{X}, \tau)$.

Theorem 4.2. Every locally closed set is $\alpha(\mathrm{gg})^*\text{-lc}^*$
Proof. Let A be a locally closed set in X. Then $\mathrm{A} = \mathrm{G} \cap \mathrm{F}$, where G is open and F is closed in X. This implies that $\mathrm{A} = \mathrm{G} \cap \mathrm{F}$, where G is $\alpha(\mathrm{gg})^*$-open and F is $\alpha(\mathrm{gg})^*$-closed in X. Thus, A is $\alpha(\mathrm{gg})^*\text{-lc}^*$ in X.

Remark 4.3. The converse of the above theorem need not be true in general as seen fromthe following example.

Example 4.4. Let $\mathrm{X} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$ with topology $\tau = \{\phi, \{\mathrm{a}\}, \{\mathrm{b}\}, \{\mathrm{a}, \mathrm{b}\}, \mathrm{X}\}$. Here $\{\mathrm{b}, \mathrm{c}\} = \{\mathrm{b}, \mathrm{c}, \mathrm{d}\} \cap \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$is $\alpha(\mathrm{gg})^*\text{-lc}^*$ but not locally closed.
Theorem 4.5. Every $\alpha(\mathrm{gg})^*\text{-lc}^*$ set in X is $\alpha(\mathrm{gg})^*$-lc.

Proof. Let A be an $\alpha(gg)^*$-lc*set in X. Then Then $A = G \cap F$, where G is $\alpha(gg)^*$-openand F is closed in X. This implies that $A = G \cap F$, where G is $\alpha(gg)^*$-open and F is $\alpha(gg)^*$-closed in X. Thus, A is $\alpha(gg)^*$-lc* in X.

Theorem 4.6. A subset A of a topological space X is $\alpha(gg)^*$-lc* if and only if $A = G \cap cl(A)$, for some $\alpha(gg)^*$-open set G.

Proof. Suppose A is $\alpha(gg)^*$-lc*. Then $A = G \cap F$, where G is $\alpha(gg)^*$-open and F is closed.Then $cl(A) = cl(G \cap F) \subseteq cl(G) \cap cl(F) \subseteq cl(F) = F$ .Then $A \subseteq G \cap cl(A) \subseteq G \cap F = A$ and hence $A = G \cap cl(A)$. Conversely, suppose that $A = G \cap cl(A)$, for some $\alpha(gg)^*$-open set G. That is, A is the intersection of an $\alpha(gg)^*$-open set and a closed set. Hence A is $\alpha(gg)^*$-lc*.

Theorem 4.7. If for a subset $K$of X, $K \cup (cl(K))^c$ is $\alpha(gg)^*$-open, then K is $\alpha(gg)^*$-lc*.

Proof. Let $K \cup \left(cl(K)\right)^c$ is $\alpha(gg)^*$-open. To prove that K is $\alpha(gg)^*$-lc*. For, $K = K \cup \phi = K \cup \left(\left(cl(K)\right)^c \cap cl(K)\right) = (K \cup (cl(K))^c) \cap (K \cup cl(K)) = (K \cup \left(cl(K)\right)^c) \cap cl(K))$, since $K \subseteq cl(K)$. So, if $K \cup (cl(K))^c$ is $\alpha(gg)^*$-open, then K is the intersection of an $\alpha(gg)^*$- open set and a closed set. Hence K is $\alpha(gg)^*$-lc*.

Theorem 4.8. If for a subset of X, the set $cl(K) - K$ is $\alpha(gg)^*$-closed, then K is $\alpha(gg)^*$-lc*.

Proof. For any subset K of X, $cl(K) - K = cl(K) \cap K^c = ((cl(K))^c \cup K)^c$.

Since $cl(K) - K$ is $\alpha(gg)^*$-closed, $K \cup (cl(K))^c$ is $\alpha(gg)^*$-open. Then by Theorem 4.7, K is $\alpha(gg)^*$-lc*.

V. $\alpha(gg)^*$-locally closed**sets

Definition 5.1. A subset A of a topological space (X, τ) is $\alpha(gg)^*$-locally closed**(briefly $\alpha(gg)^*$-lc**) set if there exists an open set G and an $\alpha(gg)^*$-closed set F such that $A = G \cap F$. The set of all $\alpha(gg)^*$-lc** sets is denoted by $\alpha(GG)^*$-LC** (X, τ).

Theorem 5.2. Every locally closed set of X is $\alpha(gg)^*$-lc**.

Proof. Let A be a locally closed set in X. Then $A = G \cap F$, where G is open and F isclosed in X. This implies that

$A = G \cap F$, where G is open and F is $\alpha(gg)^*$-closed in X. Thus, A is $\alpha(gg)^*$-lc**set in X.

Remark 5.3. The converse of the above theorem need not be true in general as seen fromthe following example.

Example 5.4. Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, c, X\}$. Here $\{c\} = \{c\} \cap \{b, c\}$ is $\alpha(gg)^*$-lc** but not locally closed.

Theorem 5.5. Every $\alpha(gg)^*$-lc** set in X is $\alpha(gg)^*$-lc.

Proof. Let A be an $\alpha(gg)^*$-lc** set in X. Then $A = G \cap F$, where G is open and F is $\alpha(gg)^*$-closed. This implies that $A = G \cap F$, where G is $\alpha(gg)^*$-open and F is $\alpha(gg)^*$-closed. Thus, A is $\alpha(gg)^*$-lc set in X.

Remark 5.6. The converse of the above theorem need not be true in general as seen fromthe following example.

Example 5.7. Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. Here $\{b\} = \{a, b\} \cap \{b, c\}$ is $\alpha(gg)^*$-lc but not $\alpha(gg)^*$-lc**.

Remark 5.8. The diagram shows the relation connecting all the locally closed sets.

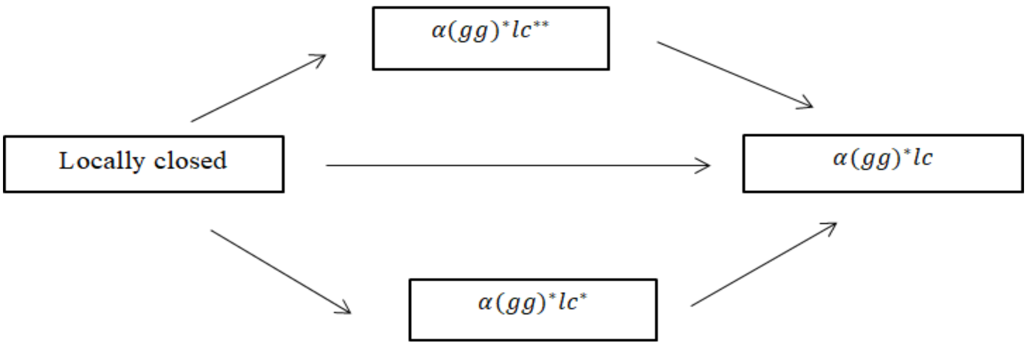

Figure: 1

Theorem 5.9.

(i) If $A \in \alpha(GG)^*LC^*(X, \tau)$ and B is closed in (X, τ) then $A \cap B \in \alpha(GG)^*LC^*(X, \tau)$.

(ii) If $A \in \alpha(GG)^*LC^{**}(X, \tau)$ and B is open in (X, τ)then $A \cap B \in \alpha(GG)^*LC^{**}(X, \tau)$.

Proof.

(i) Given that $A \in \alpha(GG)^*LC^*(X, \tau)$ and B is closed in X. Then $A = G \cap F$, where G is $\alpha(gg)^*$-

open and F is closed in X. Also given, B is closed in X. This implies that $A \cap B = (G \cap F) \cap B = G \cap (F \cap B)$, where G is $\alpha(gg)^*$-open and $F \cap B$ is closed. Thus, $A \cap B \in \alpha(GG)^*LC^*(X, \tau)$.

(ii) Given that $A \in \alpha(GG)^*LC^{**}(X, \tau)$ and B is open in X. Then $A = G \cap F$, where G is open and F is $\alpha(gg)^*$-closed in X. Also given, B is open in X. This implies that $A \cap B = (G \cap F) \cap B = (G \cap B) \cap F$, where $G \cap B$ is open and F is $\alpha(gg)^*$-closed.Thus, $A \cap B \in \alpha(GG)^*LC^{**}(X, \tau)$.

Theorem 5.10. If every $\alpha(gg)^*$-closed set is closed in (X, τ), then $\alpha(GG)^*LC(X, \tau) = \alpha(GG)^*LC^*(X, \tau)$.

Proof. Let $A \in \alpha(GG)^*LC(X, \tau)$. Then $A = G \cap F$, where G is $\alpha(gg)^*$-open and F is $\alpha(gg)^*$-closed in X. This implies that $A = G \cap F$, where G is $\alpha(gg)^*$-open and F is closed in X. Then $A \in \alpha(GG)^*LC^*(X, \tau)$. That is, $\alpha(GG)^*LC(X, \tau) \subseteq \alpha(GG)^*LC^*(X, \tau)$. From Theorem 4.5, it is clear that $\alpha(GG)^*LC^*(X, \tau) \subseteq \alpha(GG)^*LC(X, \tau)$.Hence $\alpha(GG)^*LC(X, \tau) = \alpha(GG)^*LC^*(X, \tau)$.

VI. $\alpha(gg)^*$-Locally Continuous Functions

Definition 6.1. Let $f : X \to Y$ be a function. Then f is called

(i) $\alpha(GG)^*$-LC continuous if $f^{-1}(V) \in \alpha(GG)^*LC(X)$ for each open set V of Y.
(ii) $\alpha(GG)^*$-LC* continuous if $f^{-1}(V) \in \alpha(GG)^*LC^*(X)$ for each open set V of Y.
(iii) $\alpha(GG)^*$-LC** continuous if $f^{-1}(V) \in \alpha(GG)^*LC^{**}(X)$ for each open set V of .

Theorem 6.2.

(i) Every $\alpha(gg)^*$-lc*continuous function is $\alpha(gg)^*$-lc continuous.
(ii) Every $\alpha(gg)^*$-lc**continuous function is $\alpha(gg)^*$-lc continuous.

Proof.

(i) Let f be a map that is $\alpha(gg)^*$-lc* continuous. Then for every open set V of Y, $f^{-1}(V) \in$

$\alpha(GG)^*LC^*(X)$. By Theorem 4.5, $f^{-1}(V) \in \alpha(GG)^*LC(X)$. That is, for every open set V of Y, $f^{-1}(V) \in \alpha(GG)^*LC(X)$. Therefore, f is $\alpha(gg)^*$-lc continuous.

(ii) Let f be a map that is $\alpha(gg)^*$-lc^{**} continuous. Then for every open set V of Y, $f^{-1}(V) \in \alpha(GG)^*LC^{**}(X)$. By Theorem 5.5, $f^{-1}(V) \in \alpha(GG)^*LC(X)$. That is, for every open set V of Y, $f^{-1}(V) \in \alpha(GG)^*LC(X)$. Therefore, f is $\alpha(gg)^*$-lc continuous.

Theorem 6.3. If f is a locally continuous function then f is $\alpha(gg)^*$-lc continuous (resp. $\alpha(gg)^*$-lc^* continuous, $\alpha(gg)^*$-lc^{**} continuous).

Proof. Let f be locally continuous. Then for every open set Vof Y, $f^{-1}(V) \in LC(X)$. We know that every locally closed set in X is $\alpha(gg)^*$-lc (resp. $\alpha(gg)^*$-lc^*, $\alpha(gg)^*$-lc^{**}). That is for every open set V in Y, $f^{-1}(V) \in \alpha(GG)^*LC(X)$ (resp. $f^{-1}(V) \in \alpha(GG)^*LC^*(X)$, $f^{-1}(V) \in \alpha(GG)^*LC^{**}(X)$). Therefore, f is $\alpha(gg)^*$-lc continuous (resp. .$\alpha(gg)^*$-lc^* continuous, $\alpha(gg)^*$-lc^{**} continuous).

Theorem 6.4. If X is a door space, then every map f is

(i) $\alpha(gg)^*$-lc continuous.
(ii) $\alpha(gg)^*$-lc^* continuous.
(iii) $\alpha(gg)^*$-lc^{**} continuous.

Proof.

(i) Let X be a door space and f be a map. Let A be any open set in Y. Since X is a door space, $f^{-1}(A)$ is either open or closed in X. Clearly, for every openset A in Y, $f^{-1}(A)$ is either $\alpha(gg)^*$-open or $\alpha(gg)^*$-closed in X. This implies thatfor every open set A in Y, $f^{-1}(A) = f^{-1}(A) \cap X$, where $f^{-1}(A)$ is $\alpha(gg)^*$-open and X is $\alpha(gg)^*$-closed (or) $f^{-1}(A) = X \cap f^{-1}(A)$, where X is $\alpha(gg)^*$-open and$f^{-1}(A)$ is $\alpha(gg)^*$-closed. That is, for every open set A in Y, $f^{-1}(A)$ is $\alpha(gg)^*$-lc set in X. Thus, f is $\alpha(gg)^*$-lc continuous.

(ii) For any open set A in Y, $f^{-1}(A)$ is either open or closed in X. Then for every open set A in Y, $f^{-1}(A)$ is either $\alpha(gg)^*$-open or $\alpha(gg)^*$-closed in X. This implies thatfor every open set A in Y, $f^{-1}(A) = f^{-1}(A) \cap X$, where $f^{-1}(A)$ is $\alpha(gg)^*$-

open and X is closed (or) $f^{-1}(A) = X \cap f^{-1}(A)$, where X is $\alpha(gg)^*$-open and $f^{-1}(A)$is closed. That is, for every open set A in Y, $f^{-1}(A)$ is $\alpha(gg)^*$-lc^* set in X. Thus, f is $\alpha(gg)^*$-lc^*continuous.

(iii) For any open set $\mathbf{A}$ in $\mathbf{Y}$, $\mathbf{f}^{-1}(\mathbf{A})$ is either open or closed in $\mathbf{X}$. Then for everyopen set $\mathbf{A}$ in $\mathbf{Y}$, $\mathbf{f}^{-1}(\mathbf{A})$ is either $\boldsymbol{\alpha}(\mathbf{gg})^*$-open or $\boldsymbol{\alpha}(\mathbf{gg})^*$-closed in $\mathbf{X}$. This implies that for every open set $\mathbf{A}$ in $\mathbf{Y}$, $\mathbf{f}^{-1}(\mathbf{A}) = \mathbf{f}^{-1}(\mathbf{A}) \cap \mathbf{X}$, where $\mathbf{f}^{-1}(\mathbf{A})$ is open and $\mathbf{X}$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-closed (or) $\mathbf{f}^{-1}(\mathbf{A}) = \mathbf{X} \cap \mathbf{f}^{-1}(\mathbf{A})$, where $\mathbf{X}$ is open and $\mathbf{f}^{-1}(\mathbf{A})$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-closed. That is, for every open set $\mathbf{A}$ in $\mathbf{Y}$, $\mathbf{f}^{-1}(\mathbf{A})$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-$\mathbf{lc}^{**}$ set in $\mathbf{X}$. Thus, $\mathbf{f}$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-$\mathbf{lc}^{**}$continuous.

Theorem 6.5. If $f: (X, \tau) \to (Y, \sigma)$ is $\alpha(gg)^*$-lc continuous (resp. $\alpha(gg)^*$-lc^* continuous, $\alpha(gg)^*$-lc^{**} continuous) map and $g: (Y, \sigma) \to (Z, \eta)$ is a continuous map, then $gof: (X, \tau) \to (Z, \eta)$ is $\alpha(gg)^*$-lc continuous (resp. $\alpha(gg)^*$-lc^* continuous, $\alpha(gg)^*$-lc^{**} continuous).

Proof. Let $\mathbf{g}$ be continuous and $\mathbf{f}$ be $\boldsymbol{\alpha}(\mathbf{gg})^*$-$\mathbf{lc}$ continuous. Since $\mathbf{g}$ is continuous, for every open set $\mathbf{U}$ in $\mathbf{Z}$, $\mathbf{g}^{-1}(\mathbf{U})$ is open in $\mathbf{Y}$. Since $\mathbf{f}$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-$\mathbf{lc}^*$ continuous, for every open set $\mathbf{g}^{-1}(\mathbf{U})$ in $\mathbf{Y}$, $\mathbf{f}^{-1}(\mathbf{g}^{-1}(\mathbf{U}))$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-$\mathbf{lc}$ in $\mathbf{X}$. That is, for every open set $\mathbf{g}^{-1}(\mathbf{U})$ in $\mathbf{Y}$, $(\mathbf{gof})^{-1}(\mathbf{U})$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-$\mathbf{lc}$ in X. Thus, $\mathbf{gof}: (\mathbf{X}, \boldsymbol{\tau}) \to (\mathbf{Z}, \boldsymbol{\eta})$ is $\boldsymbol{\alpha}(\mathbf{gg})^*$-$\mathbf{lc}$ continuous. Similarly, we can prove the other two.

VII. $\alpha(gg)^*$-lc irresolute maps

Definition 7.1. A map $f: (X, \tau) \to (Y, \sigma)$ is called $\alpha(gg)^*$-lc irresolute if for every $\alpha(gg)^*$-lc set V in Y, its inverse $f^{-1}(V)$ is $\alpha(gg)^*$-lc in X.

Similarly, we can define $\alpha(gg)^*$-lc^*irresolute and $\alpha(gg)^*$-lc^{**}irresolute.

Theorem 7.2. Let $f: X \to Y$ be an $\alpha(gg)^*$-irresolute map. If $K \in \alpha(GG)^*LC(Y)$ then $f^{-1}(K) \in \alpha(GG)^*LC(X)$.

Proof. Let $f: X \to Y$ be an $\alpha(gg)^*$-irresolute function. Let $K \in \alpha(GG)^*LC(Y)$. Then $= U \cap V$, where U is $\alpha(gg)^*$-open and V is $\alpha(gg)^*$-closed. This implies that $f^{-1}(K) =$

$f^{-1}(U) \cap f^{-1}(V)$. Since f is $\alpha(gg)^*$-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are $\alpha(gg)^*$-open and $\alpha(gg)^*$-closed in X respectively. Therefore, $f^{-1}(K) \in \alpha(GG)^*LC(X)$.

Theorem 7.3. Let $f: X \rightarrow Y$ be $\alpha(gg)^*$-irresolute. Then f is $\alpha(gg)^*$-lc irresolute.

Proof. It follows from previous theorem.

Remark 7.4. The converse of the above theorem need not be true in general as seen fromthe following example.

Example 7.5. Let $X = \{a, b, c\}$ with topologies $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a, c\}, Y\}$ on X and Y respectively. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ such that $f(a) = c, f(b) = a, f(c) = b$. Here f is $\alpha(gg)^*$-lc irresolute, but not $\alpha(gg)^*$-irresolute. Since for the $\alpha(gg)^*$-closed set $\{c\}$ in , its inverse image $\{a\}$ is not $\alpha(gg)^*$-closed in X.

Theorem 7.6. Any function defined on a door space is $\alpha(gg)^*$-lc irresolute.

Proof. Let $f : X \rightarrow Y$ be a function, where X is a door space and Y is any space. Let $V \in \alpha(GG)^*LC(Y)$. Then by our assumption, $f^{-1}(V)$ is either open or closed. We know that, every closed set is $\alpha(gg)^*$-closed. Now, $f^{-1}(V) = X \cap f^{-1}(V)$, where X is $\alpha(gg)^*$-open and $f^{-1}(V)$ is $\alpha(gg)^*$-closed. Therefore, $f^{-1}(V) \in \alpha(GG)^*LC(X)$. Thus, f is $\alpha(gg)^*$-lc irresolute.

Theorem 7.7. Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be any two maps. Then

(i) $gof: X \rightarrow Z$ is $\alpha(gg)^*$-lc irresolute(resp.$\alpha(gg)^*$-lc* irresolute,$\alpha(gg)^*$-lc** irresolute) if f is $\alpha(gg)^*$-lc irresolute(resp. $\alpha(gg)^*$-lc* irresolute,$\alpha(gg)^*$-lc** irresolute) and g is $\alpha(gg)^*$-lc irresolute(resp.$\alpha(gg)^*$-lc* irresolute, $\alpha(gg)^*$-lc** irresolute).
(ii) $gof: X \rightarrow Z$ is $\alpha(gg)^*$-lc continuous if f is $\alpha(gg)^*$-lc irresolute and g is $\alpha(gg)^*$-lc continuous.

Proof.

(i) Let $V \in \alpha(GG)^*LC(Z)$ (resp. $V \in \alpha(GG)^*LC^*(Z)$, $V \in \alpha(GG)^*LC^{**}(Z)$. Sinceg is $\alpha(gg)^*$-lc irresolute (resp. $\alpha(gg)^*$-lc* irresolute, $\alpha(gg)^*$-lc** irresolute),$g^{-1}(V) \in \alpha(GG)^*LC(Y)$(resp. $g^{-1}(V) \in$

$\alpha(GG)^*LC^*(Y)$, $g^{-1}(V) \in \alpha(GG)^*LC^*(Y)$). Since f is $\alpha(gg)^*$-lc irresolute (resp. $\alpha(gg)^*$-lc*irresolute,$\alpha(gg)^*$-lc**irresolute),$f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \in \alpha(GG)^*LC(X)$ (resp. $(gof)^{-1}(V) \in \alpha(GG)^*LC^*(X)$, $(gof)^{-1}(V) \in \alpha(GG)^*LC^{**}(X)$). Therefore, gof is $\alpha(gg)^*$-lc irresolute(resp. $\alpha(gg)^*$-lc* irresolute, $\alpha(gg)^*$-lc** irresolute).

(ii) Let V be any open set in Z. Since g is $\alpha(gg)^*$-lc continuous, $g^{-1}(V) \in \alpha(GG)^*LC(X)$.Since f is $\alpha(gg)^*$-lc irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \in \alpha(GG)^*LC(X)$. Therefore, gof is $\alpha(gg)^*$-lc continuous.

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