A(Gg)* - Locally Closed Sets In Topological Spaces

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Abstract
In this paper we introduce the concept of \( \alpha(gg)^{\star}\)-lc, \( \alpha(gg)^{\star}\)-lc\(^\star\) and \( \alpha(gg)^{\star}\)-lc\(^{\star}\) sets using the concept of \( \alpha(gg)^{\star}\)-open and \( \alpha(gg)^{\star}\)-closed sets. Also, we present some of the weaker forms of locally continuous functions.

Keywords: \( \alpha(gg)^{\star}\)-lc sets, \( \alpha(gg)^{\star}\)-lc\(^{\star}\) sets, \( \alpha(gg)^{\star}\)-lc\(^{\star}\) sets, \( \alpha(gg)^{\star}\)-lc continuous functions

I. INTRODUCTION

In 1921, Kuratowski and Sierpinski [1] introduced the notion of locally closed sets in topological spaces. In 1989, Ganster and Reilly [5] introduced the concept of LC- continuous and LC- irresolute maps using locally closed sets. Later on, many researchers have investigated on this topic and introduced different types of locally closed sets. In 2022, authors [2] introduced \( \alpha(gg)^{\star}\)- closed sets in topological spaces. In this paper, authors put forth the concept of \( \alpha(gg)^{\star}\)-lc, \( \alpha(gg)^{\star}\)-lc\(^\star\) and \( \alpha(gg)^{\star}\)-lc\(^{\star}\) sets using the concept of \( \alpha(gg)^{\star}\)-open and \( \alpha(gg)^{\star}\)-closed sets. Later on, we discussed about \( \alpha(gg)^{\star}\)-LC continuous (resp. \( \alpha(gg)^{\star}\)-lc irresolute) functions and some of its basic properties are examined

II. PRELIMINARIES

Throughout this paper \((X, \tau)\) represents the topological spaces on which no separation axioms are assumed unless otherwise mentioned. \( A \) being a subset of a topological space \((X, \tau)\), \( \text{cl}(A) \), \( \text{int}(A) \) denote the closure of \( A \) and interior of \( A \) respectively.

Definition 2.1 [2] A set \( A \) of a topological space \((X, \tau)\) is called alpha generalization of generalized star closed (briefly \( \alpha(gg)^{\star}\)-
closed) if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $(gg)^*\text{-open}$ in $(X, \tau)$.

**Definition 2.2.** [5] A set $A$ of a topological space $(X, \tau)$ is called locally closed if $A = G \cap F$, where $G$ is open and $F$ is closed.

**Definition 2.4.** A function $f: X \rightarrow Y$ is called

(i) $(gg)^*\text{-continuous}$ [3] if $f^{-1}(V)$ is $(gg)^*\text{-closed}$ in $X$ for every closed subset $V$ of $Y$.

(ii) $(gg)^*\text{-irresolute}$ if $f^{-1}(V)$ is $(gg)^*\text{-closed}$ in $(X, \tau)$ for every $(gg)^*\text{-closed}$ subset $V$ of $(Y, \sigma)$.

**Definition 2.5.** [4] A topological space $(X, \tau)$ is said to be a door space iff every subset of $X$ is either open or closed.

**Lemma 2.6.** [2] Every closed set is $(gg)^*\text{-closed}$.

### III. MAIN RESULTS

**Definition 3.1.** A set $A$ of a topological space $(X, \tau)$ is called $(gg)^*\text{-locally closed}$ (briefly $(gg)^*\text{-lc}$) if $A = G \cap F$, where $G$ is an $(gg)^*\text{-open}$ set and $F$ is an $(gg)^*\text{-closed}$ set.

**Theorem 3.2.**

(i) Every $(gg)^*\text{-open}$ set in $X$ is $(gg)^*\text{-lc}$.

(ii) Every $(gg)^*\text{-closed}$ set in $X$ is $(gg)^*\text{-lc}$.

**Proof.**

(i) Let $A$ be an $(gg)^*\text{-open}$ set in $X$. Then $A = A \cap X$, where $A$ is $(gg)^*\text{-open}$ and $X$ is $(gg)^*\text{-closed}$. Thus $A$ is $(gg)^*\text{-lc}$.

(ii) Let $A$ be an $(gg)^*\text{-closed}$ set in $X$. Then $A = X \cap A$, where $X$ is $(gg)^*\text{-open}$ and $A$ is $(gg)^*\text{-closed}$. Thus $A$ is $(gg)^*\text{-lc}$.

**Remark 3.3.** The converse of the above theorem need not be true in general as seen from the following example.

**Example 3.4.** Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

(i) $\{c, d\} = X \cap \{c, d\}$. Here $\{c, d\}$ is $(gg)^*\text{-lc}$ but not $(gg)^*\text{-open}$.

(ii) $\{a\} = \{a\} \cap \{a, d\}$. Here $\{a\}$ is $(gg)^*\text{-lc}$ but not $(gg)^*\text{-closed}$.

**Theorem 3.5.** In a topological space $(X, \tau)$

Every open set is $(gg)^*\text{-lc}$.

(i) Every closed set is $(gg)^*\text{-lc}$.
Proof.

(i) Let $A$ be an open set in $X$. Then $A$ is $\alpha(gg)^*$-open in $X$. Thus, $A$ is $\alpha(gg)^*$-lc in $X$.

(ii) Let $A$ be a closed set in $X$. Then $A$ is $\alpha(gg)^*$-closed in $X$. Thus, $A$ is $\alpha(gg)^*$-lc in $X$.

Remark 3.6. The converse of the above theorem need not be true in general as seen from the following example.

Example 3.7. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

(i) $\{b, c\} = \{a, b, c\} \cap \{b, c, d\}$. Here $\{b, c\}$ is $\alpha(gg)^*$-lc but not open.

(ii) $\{a, b\} = \{a, b, c\} \cap \{a, b, d\}$. Here $\{a, b\}$ is $\alpha(gg)^*$-lc but not closed.

Theorem 3.8. Every locally closed set is $\alpha(gg)^*$-locally closed.

Proof. Let $A$ be a locally closed subset of $X$. Then $A = G \cap F$, where $G$ is open and $F$ is closed. That is, $A = G \cap F$, where $G$ is $\alpha(gg)^*$-open and $F$ is $\alpha(gg)^*$-closed. Thus, $A$ is $\alpha(gg)^*$-locally closed in $X$.

Remark 3.9. The converse of the above theorem need not be true in general as seen from the following example.

Example 3.10. Let $X = \{a, b, c\}$ with topology $\tau = \{\phi, \{a\}, \{b, c\}\}$. Then $\{c\} = \{a, c\} \cap \{b, c\}$ is $\alpha(gg)^*$-locally closed but not locally closed.

Theorem 3.11. A subset $A$ of $X$ is $\alpha(gg)^*$-lc if and only if its complement, $A^c$ is the union of an $\alpha(gg)^*$-open set and an $\alpha(gg)^*$-closed set.

Proof. Let $A$ be an $\alpha(gg)^*$-lc set in $(X, \tau)$. Then $A = G \cap F$, where $G$ is $\alpha(gg)^*$-open and $F$ is $\alpha(gg)^*$-closed. This implies that, $A^c = (G \cap F)^c = F^c \cup G^c$, where $F^c$ is $\alpha(gg)^*$-open and $G^c$ is $\alpha(gg)^*$-closed. Conversely, assume that $A^c$ is the union of an $\alpha(gg)^*$-open set and an $\alpha(gg)^*$-closed set. That is, $A^c = G \cup F$, where $G$ is $\alpha(gg)^*$-open and $F$ is $\alpha(gg)^*$-closed. This implies that $(A^c)^c = (G \cup F)^c$. That is, $A = F^c \cap G^c$, where $F^c$ is $\alpha(gg)^*$-open and $G^c$ is $\alpha(gg)^*$-closed. Thus, $A$ is $\alpha(gg)^*$-lc set in $X$.

Theorem 3.12. Let $f: X \to Y$ be a continuous function. If $K$ is locally closed subset of $Y$, then $f^{-1}(K)$ is $\alpha(gg)^*$-lc in $X$.

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**Proof.** Let $K$ be a locally closed subset of $Y$. Then $K = G \cap F$, where $G$ is open and $F$ is closed. This implies that $f^{-1}(K) = f^{-1}(G \cap F) = f^{-1}(G) \cap f^{-1}(F)$, where $f^{-1}(G)$ is open and $f^{-1}(F)$ is closed. That is, $f^{-1}(K) = f^{-1}(G) \cap f^{-1}(F)$, where $f^{-1}(G)$ is $\alpha(gg)^*$-open and $f^{-1}(F)$ is $\alpha(gg)^*$-closed. Thus, $f^{-1}(K)$ is $\alpha(gg)^*$-lc in $X$.

**Theorem 3.13.** For a subset $A$ of a topological space $X$

(i) $A$ is $\alpha(gg)^*$-lc.
(ii) $A = U \cap \alpha(gg)^*\text{cl}(A)$ for some $\alpha(gg)^*$-open set $U$.

**Proof.**

(i)$\Rightarrow$(ii)

Suppose $A$ is $\alpha(gg)^*$-lc. Then $A = U \cap F$, where $U$ is $\alpha(gg)^*$-open and $F$ is $\alpha(gg)^*$-closed. That is, $\alpha(gg)^*\text{cl}(A) = \alpha(gg)^*\text{cl}(U \cap F) \subseteq \alpha(gg)^*\text{cl}(U) \cap \alpha(gg)^*\text{cl}(F) \subseteq \alpha(gg)^*\text{cl}(F) = F$. Thus, $\alpha(gg)^*\text{cl}(A) \subseteq F$. Then $A \subseteq U \cap \alpha(gg)^*\text{cl}(A) \subseteq U \cap F = A$. Hence $A = U \cap \alpha(gg)^*\text{cl}(A)$.

(ii)$\Rightarrow$(i)

Suppose that $A = U \cap \alpha(gg)^*\text{cl}(A)$ for some $\alpha(gg)^*$-open set $U$. Since $\text{cl}(A)$ is closed, by Lemma.2.6 the theorem follows.

**IV. $\alpha(gg)^*$-locally closed* sets**

**Definition 4.1.** A subset $A$ of a topological space $(X, \tau)$ is an $\alpha(gg)^*$-locally closed* (briefly $\alpha(gg)^*$-lc*) set if there exists an $\alpha(gg)^*$-open set $G$ and a closed set $F$ in $X$ such that $A = G \cap F$. The set of all $\alpha(gg)^*$-lc* subsets of $(X, \tau)$ is denoted by $\alpha(GG)^*$-LC*$(X, \tau)$.

**Theorem 4.2.** Every locally closed set is $\alpha(gg)^*$-lc*

**Proof.** Let $A$ be a locally closed set in $X$. Then $A = G \cap F$, where $G$ is open and $F$ is closed in $X$. This implies that $A = G \cap F$, where $G$ is $\alpha(gg)^*$-open and $F$ is $\alpha(gg)^*$-closed in $X$. Thus, $A$ is $\alpha(gg)^*$-lc* in $X$.

**Remark 4.3.** The converse of the above theorem need not be true in general as seen from the following example.

**Example 4.4.** Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Here $\{b, c\} = \{b, c, d\} \cap \{a, b, c\}$ is $\alpha(gg)^*$-lc* but not locally closed.

**Theorem 4.5.** Every $\alpha(gg)^*$-lc* set in $X$ is $\alpha(gg)^*$-lc.
Proof. Let A be an $\alpha(gg)^*-lc^*$ set in X. Then $A = G \cap F$, where G is $\alpha(gg)^*$-open and F is closed in X. This implies that $A = G \cap F$, where G is $\alpha(gg)^*$-open and F is $\alpha(gg)^*$-closed in X. Thus, A is $\alpha(gg)^*-lc^*$ in X.

**Theorem 4.6.** A subset A of a topological space X is $\alpha(gg)^*-lc^*$ if and only if $A = G \cap cl(A)$, for some $\alpha(gg)^*$-open set G.

Proof. Suppose A is $\alpha(gg)^*-lc^*$. Then $A = G \cap F$, where G is $\alpha(gg)^*$-open and F is closed. Then $cl(A) = cl(G \cap F) \subseteq cl(G) \cap cl(F) \subseteq cl(F) = F$. Then $A \subseteq G \cap cl(A) \subseteq G \cap F = A$ and hence $A = G \cap cl(A)$. Conversely, suppose that $A = G \cap cl(A)$, for some $\alpha(gg)^*$-open set G. That is, A is the intersection of an $\alpha(gg)^*$-open set and a closed set. Hence A is $\alpha(gg)^*-lc^*$.

**Theorem 4.7.** If for a subset K of X, $K \cup (cl(K))^c$ is $\alpha(gg)^*$-open, then K is $\alpha(gg)^*-lc^*$.

Proof. Let $K \cup (cl(K))^c$ is $\alpha(gg)^*$-open. To prove that K is $\alpha(gg)^*-lc^*$. For, $K = K \cup \phi = K \cup ((cl(K))^c \cap cl(K)) = (K \cup (cl(K))^c) \cap (K \cup cl(K)) = (K \cup (cl(K))^c) \cap cl(K)$, since $K \subseteq cl(K)$. So, if $K \cup (cl(K))^c$ is $\alpha(gg)^*$-open, then K is the intersection of an $\alpha(gg)^*$-open set and a closed set. Hence K is $\alpha(gg)^*-lc^*$.

**Theorem 4.8.** If for a subset of X, the set $cl(K) - K$ is $\alpha(gg)^*$-closed, then K is $\alpha(gg)^*-lc^*$.

Proof. For any subset K of X, $cl(K) - K = cl(K) \cap K^c = ((cl(K))^c \cup K)^c$.

Since $cl(K) - K$ is $\alpha(gg)^*$-closed, $K \cup (cl(K))^c$ is $\alpha(gg)^*$-open. Then by Theorem 4.7, K is $\alpha(gg)^*-lc^*$.

V. $\alpha(gg)^*$-locally closed** sets

**Definition 5.1.** A subset A of a topological space $(X, \tau)$ is $\alpha(gg)^*$-locally closed** (briefly $\alpha(gg)^*-lc^*$) set if there exists an open set G and an $\alpha(gg)^*$-closed set F such that $A = G \cap F$. The set of all $\alpha(gg)^*-lc^*$ sets is denoted by $\alpha(GG)^*-LC^*$ $(X, \tau)$.

**Theorem 5.2.** Every locally closed set of X is $\alpha(gg)^*-lc^*$.

Proof. Let A be a locally closed set in X. Then $A = G \cap F$, where G is open and F is closed in X. This implies that
A = G ∩ F, where G is open and F is α(gg)*-closed in X. Thus, A is α(gg)*-lc** set in X.

**Remark 5.3.** The converse of the above theorem need not be true in general as seen from the following example.

**Example 5.4.** Let X = {a, b, c} with topology τ = {ϕ, c, X}. Here {c} = {c} ∩ {b, c} is α(gg)*-lc but not locally closed.

**Theorem 5.5.** Every α(gg)*-lc** set in X is α(gg)*-lc.

**Proof.** Let A be an α(gg)*-lc** set in X. Then A = G ∩ F, where G is open and F is α(gg)*-closed. This implies that A = G ∩ F, where G is α(gg)*-open and F is α(gg)*-closed. Thus, A is α(gg)*-lc set in X.

**Remark 5.6.** The converse of the above theorem need not be true in general as seen from the following example.

**Example 5.7.** Let X = {a, b, c} with topology τ = {ϕ, {a}, {b, c}, X}. Here {b} = {a, b} ∩ {b, c} is α(gg)*-lc but not α(gg)*-lc**.

**Remark 5.8.** The diagram shows the relation connecting all the locally closed sets.

![Diagram](image)

Figure: 1

**Theorem 5.9.**

(i) If A ∈ α(GG)*LC*(X, τ) and B is closed in (X, τ) then A ∩ B ∈ α(GG)*LC*(X, τ).

(ii) If A ∈ α(GG)*LC***(X, τ) and B is open in (X, τ) then A ∩ B ∈ α(GG)*LC***(X, τ).

**Proof.**

(i) Given that A ∈ α(GG)*LC*(X, τ) and B is closed in X. Then A = G ∩ F, where G is α(gg)*-
open and F is closed in X. Also given, B is closed in X. This implies that 
\[ A \cap B = (G \cap F) \cap B = G \cap (F \cap B), \]
where G is \( \alpha(gg)^* \)-open and F \( \cap \) B is closed. Thus, 
\[ A \cap B \in \alpha(GG)^* \text{LC}^*(X, \tau). \]
(ii) Given that \( A \in \alpha(GG)^* \text{LC}^*(X, \tau) \) and B is open in X. 
Then \( A = G \cap F \), where G is open and F is \( \alpha(gg)^* \)-closed in X. 
Also given, B is open in X. This implies that 
\[ A \cap B = (G \cap F) \cap B = (G \cap B) \cap F, \]
where G \( \cap \) B is open and F is \( \alpha(gg)^* \)-closed. Thus, 
\[ A \cap B \in \alpha(GG)^* \text{LC}^*(X, \tau). \]

**Theorem 5.10.** If every \( \alpha(gg)^* \)-closed set is closed in 
\( (X, \tau) \), then 
\[ \alpha(GG)^* \text{LC}(X, \tau) = \alpha(GG)^* \text{LC}^*(X, \tau). \]

**Proof.** Let \( A \in \alpha(GG)^* \text{LC}(X, \tau) \). Then \( A = G \cap F \), 
where G is \( \alpha(gg)^* \)-open and F is \( \alpha(gg)^* \)-closed in X. 
This implies that \( A = G \cap F \), where G is \( \alpha(gg)^* \)-open 
and F is closed in X. Then \( A \in \alpha(GG)^* \text{LC}^*(X, \tau) \). 
That is, \( \alpha(GG)^* \text{LC}(X, \tau) \subseteq \alpha(GG)^* \text{LC}^*(X, \tau) \). From 
Theorem 4.5, it is clear that \( \alpha(GG)^* \text{LC}^*(X, \tau) \subseteq 
\alpha(GG)^* \text{LC}^*(X, \tau) \). Hence 
\[ \alpha(GG)^* \text{LC}(X, \tau) = \alpha(GG)^* \text{LC}^*(X, \tau). \]

**VI. \( \alpha(gg)^* \)-Locally Continuous Functions**

**Definition 6.1.** Let \( f : X \to Y \) be a function. Then \( f \) 
is called 

(i) \( \alpha(GG)^* \)-\text{LC continuous} if \( f^{-1}(V) \in \alpha(GG)^* \text{LC} \) 
\( (X) \) for each open set \( V \) of \( Y \).

(ii) \( \alpha(GG)^* \)-\text{LC}^* continuous if \( f^{-1}(V) \in \alpha(GG)^* \text{LC}^* \) 
\( (X) \) for each open set \( V \) of \( Y \).

(iii) \( \alpha(GG)^* \)-\text{LC}^{**} continuous if \( f^{-1}(V) \in \alpha(GG)^* \text{LC}^{**} \) 
\( (X) \) for each open set \( V \) of .

**Theorem 6.2.**

(i) Every \( \alpha(gg)^* \)-\text{lc}^{**} continuous function is \( \alpha(gg)^* \)-\text{lc} 
continuous.

(ii) Every \( \alpha(gg)^* \)-\text{lc}^{**} continuous function is \( \alpha(gg)^* \)-\text{lc} 
continuous.

**Proof.**

(i) Let \( f \) be a map that is \( \alpha(gg)^* \)-\text{lc}^{**} continuous. Then 
for every open set \( V \) of \( Y \), \( f^{-1}(V) \in \)
\(\alpha(GG)^{\ast}\text{LC}^{\ast}(X)\). By Theorem 4.5, \(f^{-1}(V) \in \alpha(GG)^{\ast}\text{LC}(X)\). That is, for every open set \(V\) of \(Y\), 
\(f^{-1}(V) \in \alpha(GG)^{\ast}\text{LC}^{\ast}(X)\). Therefore, \(f\) is \(\alpha(gg)^{\ast}\text{-lc}^\ast\) continuous.

(ii) Let \(f\) be a map that is \(\alpha(gg)^{\ast}\text{-lc}^{\ast\ast}\) continuous. Then for every open set \(V\) of \(Y\), 
\(f^{-1}(V) \in \alpha(GG)^{\ast}\text{LC}^{\ast\ast}(X)\). By Theorem 5.5, 
\(f^{-1}(V) \in \alpha(GG)^{\ast}\text{LC}(X)\). That is, for every open set \(V\) of \(Y\), 
\(f^{-1}(V) \in \alpha(GG)^{\ast}\text{LC}(X)\). Therefore, \(f\) is \(\alpha(gg)^{\ast}\text{-lc}^\ast\) continuous.

**Theorem 6.3.** If \(f\) is a locally continuous function then \(f\) is \(\alpha(gg)^{\ast}\text{-lc}^\ast\) continuous (resp. \(\alpha(gg)^{\ast}\text{-lc}^{\ast\ast}\) continuous, \(\alpha(gg)^{\ast}\text{-lc}^{\ast\ast\ast}\) continuous).

**Proof.** Let \(f\) be locally continuous. Then for every open set \(V\) of \(Y\), 
\(f^{-1}(V) \in \text{LC}(X)\). We know that every locally closed set in \(X\) is \(\alpha(gg)^{\ast}\text{-lc}^\ast\) (resp. \(\alpha(gg)^{\ast}\text{-lc}^{\ast\ast}\), \(\alpha(gg)^{\ast}\text{-lc}^{\ast\ast\ast}\)). That is, for every open set \(V\) in \(Y\), 
\(f^{-1}(V) \in \alpha(GG)^{\ast}\text{LC}(X)\) (resp. 
\(f^{-1}(V) \in \alpha(GG)^{\ast}\text{LC}^{\ast\ast}(X)\), 
\(f^{-1}(V) \in \alpha(GG)^{\ast}\text{LC}^{\ast\ast\ast}(X)\)). Therefore, \(f\) is \(\alpha(gg)^{\ast}\text{-lc}^\ast\) continuous (resp. \(\alpha(gg)^{\ast}\text{-lc}^{\ast\ast}\) continuous, \(\alpha(gg)^{\ast}\text{-lc}^{\ast\ast\ast}\) continuous).

**Theorem 6.4.** If \(X\) is a door space, then every map \(f\) is

(i) \(\alpha(gg)^{\ast}\text{-lc}^\ast\) continuous.
(ii) \(\alpha(gg)^{\ast}\text{-lc}^{\ast\ast}\) continuous.
(iii) \(\alpha(gg)^{\ast}\text{-lc}^{\ast\ast\ast}\) continuous.

**Proof.**

(i) Let \(X\) be a door space and \(f\) be a map. Let \(A\) be any open set in \(Y\). Since \(X\) is a door space, \(f^{-1}(A)\) is either open or closed in \(X\). Clearly, for every open set \(A\) in \(Y\), 
\(f^{-1}(A)\) is either \(\alpha(gg)^{\ast}\)-open or \(\alpha(gg)^{\ast}\) closed in \(X\). This implies that for every open set \(A\) in \(Y\), 
\(f^{-1}(A) = f^{-1}(A) \cap X\), where \(f^{-1}(A)\) is \(\alpha(gg)^{\ast}\)open and \(X\) is \(\alpha(gg)^{\ast}\)closed (or) \(f^{-1}(A) = X \cap f^{-1}(A)\), 
where \(X\) is \(\alpha(gg)^{\ast}\)open and \(f^{-1}(A)\) is \(\alpha(gg)^{\ast}\)closed. That is, for every open set \(A\) in \(Y\), 
\(f^{-1}(A)\) is \(\alpha(gg)^{\ast}\text{-lc}\) set in \(X\). Thus, \(f\) is \(\alpha(gg)^{\ast}\text{-lc}^\ast\) continuous.

(ii) For any open set \(A\) in \(Y\), \(f^{-1}(A)\) is either open or closed in \(X\). Then for every open set \(A\) in \(Y\), 
\(f^{-1}(A)\) is either \(\alpha(gg)^{\ast}\)open or \(\alpha(gg)^{\ast}\) closed in \(X\). This implies that for every open set \(A\) in \(Y\), 
\(f^{-1}(A) = f^{-1}(A) \cap X\), where \(f^{-1}(A)\) is \(\alpha(gg)^{\ast}\)
open and $X$ is closed (or) $f^{-1}(A) = X \cap f^{-1}(A)$, where $X$ is $\alpha(gg)^*$-open and $f^{-1}(A)$ is closed. That is, for every open set $A$ in $Y$, $f^{-1}(A)$ is $\alpha(gg)^*$-closed in $X$. Thus, $f$ is $\alpha(gg)^*$-lc"-continuous.

(iii) For any open set $A$ in $Y$, $f^{-1}(A)$ is either open or closed in $X$. Then for every open set $A$ in $Y$, $f^{-1}(A)$ is either $\alpha(gg)^*$-open or $\alpha(gg)^*$-closed in $X$. This implies that for every open set $A$ in $Y$, $f^{-1}(A) = f^{-1}(A) \cap X$, where $f^{-1}(A)$ is open and $X$ is $\alpha(gg)^*$-closed (or) $f^{-1}(A) = X \cap f^{-1}(A)$, where $X$ is open and $f^{-1}(A)$ is $\alpha(gg)^*$-closed. That is, for every open set $A$ in $Y$, $f^{-1}(A)$ is $\alpha(gg)^*$-lc"-continuous.

**Theorem 6.5.** If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\alpha(gg)^*$-lc continuous (resp. $\alpha(gg)^*$-lc" continuous, $\alpha(gg)^*$-lc** continuous) map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is a continuous map, then $gof : (X, \tau) \rightarrow (Z, \eta)$ is $\alpha(gg)^*$-lc continuous (resp. $\alpha(gg)^*$-lc" continuous, $\alpha(gg)^*$-lc** continuous).

**Proof.** Let $g$ be continuous and $f$ be $\alpha(gg)^*$-lc continuous. Since $g$ is continuous, for every open set $U$ in $Z$, $g^{-1}(U)$ is open in $Y$. Since $f$ is $\alpha(gg)^*$-lc" continuous, for every open set $g^{-1}(U)$ in $Y$, $f^{-1}(g^{-1}(U))$ is $\alpha(gg)^*$-lc in $X$. That is, for every open set $g^{-1}(U)$ in $Y$, $(gof)^{-1}(U)$ is $\alpha(gg)^*$-lc in $X$. Thus, $gof : (X, \tau) \rightarrow (Z, \eta)$ is $\alpha(gg)^*$-lc continuous. Similarly, we can prove the other two.

VII. $\alpha(gg)^*$-lc" irresolute maps

**Definition 7.1.** A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $\alpha(gg)^*$-lc irresolute if for every $\alpha(gg)^*$-lc set $V$ in $Y$, its inverse $f^{-1}(V)$ is $\alpha(gg)^*$-lc in $X$.

Similarly, we can define $\alpha(gg)^*$-lc" irresolute and $\alpha(gg)^*$-lc** irresolute.

**Theorem 7.2.** Let $f : X \rightarrow Y$ be an $\alpha(gg)^*$-irresolute map. If $K \in \alpha(GG)^*LC(Y)$ then $f^{-1}(K) \in \alpha(GG)^*LC(X)$.

**Proof.** Let $f : X \rightarrow Y$ be an $\alpha(gg)^*$-irresolute function. Let $K \in \alpha(GG)^*LC(Y)$. Then $= U \cap V$, where $U$ is $\alpha(gg)^*$-open and $V$ is $\alpha(gg)^*$-closed. This implies that $f^{-1}(K) =$.
f^{-1}(U) \cap f^{-1}(V). Since f is \(\alpha gg^*\)-irresolute, \(f^{-1}(U)\) and 
\(f^{-1}(V)\) are \(\alpha gg^*\)-open and \(\alpha gg^*\)-closed in \(X\) respectively. Therefore, \(f^{-1}(K) \in \alpha (GG)^*LC(X)\).

**Theorem 7.3.** Let \(f: X \to Y\) be \(\alpha gg^*\)-irresolute. Then \(f\) is \(\alpha gg^*\)-lc irresolute.

**Proof.** It follows from previous theorem.

**Remark 7.4.** The converse of the above theorem need not be true in general as seen from the following example.

**Example 7.5.** Let \(X = \{a, b, c\}\) with topologies \(\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}\) and \(\sigma = \{\phi, \{a, c\}, Y\}\) on \(X\) and \(Y\) respectively. Define a function \(f: (X, \tau) \to (Y, \sigma)\) such that \(f(a) = c, f(b) = a, f(c) = b\). Here \(f\) is \(\alpha (gg)^*\)-lc irresolute, but not \(\alpha (gg)^*\)-irresolute. Since the \(\alpha (gg)^*\)-closed set \(\{c\}\) in \(X\), its inverse image \(\{a\}\) is not \(\alpha (gg)^*\)-closed in \(X\).

**Theorem 7.6.** Any function defined on a door space is \(\alpha (gg)^*\)-lc irresolute.

**Proof.** Let \(f: X \to Y\) be a function, where \(X\) is a door space and \(Y\) is any space. Let \(V \in \alpha (GG)^*LC(Y)\). Then by our assumption, \(f^{-1}(V)\) is either open or closed. We know that, every closed set is \(\alpha (gg)^*\)-closed. Now, \(f^{-1}(V) = X \cap f^{-1}(V),\) where \(X\) is \(\alpha (gg)^*\)-open and \(f^{-1}(V)\) is \(\alpha (gg)^*\)-closed. Therefore, \(f^{-1}(V) \in \alpha (GG)^*LC(X)\). Thus, \(f\) is \(\alpha (gg)^*\)-lc irresolute.

**Theorem 7.7.** Let \(f: X \to Y\) and \(g: Y \to Z\) be any two maps. Then

(i) \(gof:X \to Z\) is \(\alpha (gg)^*\)-lc irresolute (resp. \(\alpha (gg)^*\)-lc** irresolute, \(\alpha (gg)^*\)-lc** irresolute) if \(f\) is \(\alpha (gg)^*\)-lc irresolute (resp. \(\alpha (gg)^*\)-lc** irresolute, \(\alpha (gg)^*\)-lc** irresolute) and \(g\) is \(\alpha (gg)^*\)-lc irresolute (resp. \(\alpha (gg)^*\)-lc** irresolute, \(\alpha (gg)^*\)-lc** irresolute).

(ii) \(gof:X \to Z\) is \(\alpha (gg)^*\)-lc continuous if \(f\) is \(\alpha (gg)^*\)-lc irresolute and \(g\) is \(\alpha (gg)^*\)-lc continuous.

**Proof.**

(i) Let \(V \in \alpha (GG)^*LC(Z)\) (resp. \(V \in \alpha (GG)^*LC^*(Z)\), \(V \in \alpha (GG)^*LC^{**}(Z)\). Since \(g\) is \(\alpha (gg)^*\)-lc irresolute (resp. \(\alpha (gg)^*\)-lc** irresolute, \(\alpha (gg)^*\)-lc** irresolute), \(g^{-1}(V) \in \alpha (GG)^*LC(Y)\) (resp. \(g^{-1}(V) \in \alpha (GG)^*LC^*(Y)\), \(g^{-1}(V) \in \alpha (GG)^*LC^{**}(Y)\). Therefore, \(gof(X) \in \alpha (GG)^*LC(Z)\) (resp. \(gof(X) \in \alpha (GG)^*LC^*(Z)\), \(gof(X) \in \alpha (GG)^*LC^{**}(Z)\).
\(\alpha(GG)^*\text{LC}^*(Y), \quad g^{-1}(V) \in \alpha(GG)^*\text{LC}^*(Y)\). Since \(f\) is \(\alpha(gg)^*-\text{lc irresolute}\) (resp. \(\alpha(gg)^*\text{-lc'' irresolute}\)), \(f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)\in \alpha(GG)^*\text{LC}(X)\) (resp. \((gof)^{-1}(V) \in \alpha(GG)^*\text{LC}^*(X)\)). Therefore, \(gof\) is \(\alpha(gg)^*-\text{lc irresolute}\) (resp. \(\alpha(gg)^*\text{-lc'' irresolute}\)).

(ii) Let \(V\) be any open set in \(Z\). Since \(g\) is \(\alpha(gg)^*-\text{lc continuous}\), \(g^{-1}(V) \in \alpha(GG)^*\text{LC}(X)\). Since \(f\) is \(\alpha(gg)^*-\text{lc irresolute}\), \(f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)\in \alpha(GG)^*\text{LC}(X)\). Therefore, \(gof\) is \(\alpha(gg)^*-\text{lc continuous}\).

REFERENCES


