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STRONGLY AND PERFECTLY $I\alpha_G^{\wedge}$-CONTINUOUS FUNCTION IN INTUITIONISTIC TOPOLOGICAL SPACES

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ABSTRACT

The concept of intuitionistic sets and intuitionistic points in topological spaces was first introduced by Coker. The purpose of this paper is to introduce a mapping of strongly $I\alpha_g^{\wedge}$-continuous and perfectly $I\alpha_g^{\wedge}$-continuous function in intuitionistic topological spaces and analyze its relations with other existing intuitionistic functions.

Keywords: strongly $I\alpha_g^{\wedge}$-continuous and perfectly $I\alpha_g^{\wedge}$-continuous.

INTRODUCTION

Coker [1] introduced the concept of intuitionistic sets and intuitionistic points in 1996. [3] J.G.Lee, P.K.Lim, J.H.Kim, K.Hur introduced intuitionistic continuous, closed and open mappings in 2017. [4] J.Arul Jesti and K.Heartlin introduced the concept of alpha^generalized closed sets in intuitionistic topological spaces and discuss some properties related to $I\alpha_g^{\wedge}$-closed set in intuitionistic topological spaces. [5] J.Arul Jesti and K.Heartlin On $I\alpha_g^{\wedge}$-continuous function In intuitionistic topological spaces.The purpose of this paper is to develop some $I\alpha_g^{\wedge}$-continuous function in intuitionistic topological spaces and also study its relations with some of existing intuitionistic relations.

2 PRELIMINARIES

Definition 2.1 [1]: Let $\mathcal{H}$ be a non-empty set. An intuitionistic set (IS for short) A is an object having the form A=$<\mathcal{H}, A_1, A_2>$ Where A_1, A_2 are subsets of $\mathcal{H}$ satisfying $A_1 \cap A_2 = \varphi$. The set A_1 is called the set of members of A, while A_2 is called set of non members of

A.

Definition 2.2 [1]: Let $\mathcal{H}$ be a non-empty set and A and B are intuitionistic set in the form $A =< \mathcal{H}, A_1, A_2 >$, $B =< \mathcal{H}, B_1, B_2 >$ respectively. Then

a) $A \subseteq B$ iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$

b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$

c) $A^C = < \mathcal{H}, A_2, A_1 >$

d) $A - B = A \cap B^C$

e) $\varphi = < \mathcal{H}, \varphi, \mathcal{H} >, \mathcal{H} =< \mathcal{H}, \mathcal{H}, \varphi >$

f) $A \cup B = < \mathcal{H}, A_1 \cup B_1, A_2 \cap B_2 >$

g) $A \cap B = < \mathcal{H}, A_1 \cap B_1, A_2 \cup B_2 >$.

Definition 2.3 [3]: A subset A of ($\mathcal{H}$,Iτ) is called an **intuitionistic alpha ^ generalized closed (**briefly **$I\alpha_g^{\wedge}$-closed)** if Igcl(A) ⊆ U, whenever A⊆ U and U is Iα-open in $\mathcal{H}$. We denote the family of all $I\alpha_g^{\wedge}$-closed sets in space $\mathcal{H}$ by $I\alpha_g^{\wedge}C(\mathcal{H})$.

Definition 2.4 [6]: Let $(\mathcal{H}, \tau_1)$ and (Y, τ_2) be two ITS's and $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ be a function. Then f is said to be intuitionistic continuous iff the pre image of each Is in τ_2 is an Is in τ_1.

3. Strongly $I\alpha_g^{\wedge}$-Continuous

This section we define the concepts of strongly $I\alpha_g^{\wedge}$-continuous and some results are discussed.

Definition 3.1: A function $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is called a **strongly $I\alpha_g^{\wedge}$-continuous function** if the inverse image of every $I\alpha_g^{\wedge}$-open set in $(Y, I\tau_\theta)$ is I-open in $(\mathcal{H}, I\tau_\mu)$.

Example 3.2: Let $\mathcal{H} = \{a, b\}$ and $I\tau_\mu = \{\mathcal{H}, \varphi, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \{b\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{a\}, \{b\} >, \mathcal{A}_3 = < \mathcal{H}, \{a\}, \varphi >, \mathcal{A}_4 =< \mathcal{H}, \varphi, \varphi >$. Let $Y = \{a, b\}$ and $I\tau_\theta$={Y, ϕ, $< Y, \varphi, \{b\} >, < Y, \{a\}, \{b\} >, < Y, \{a\}, \varphi >$ } $= I\alpha_g^{\wedge} - O(Y)$. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as $f(a) = a, f(b) = b$. Then $f^{-1}(< Y, \varphi, \{b\} >) =< \mathcal{H}, \varphi, \{b\} >, f^{-1}(< Y, \{a\}, \{b\} >) =< \mathcal{H}, \{a\}, \{b\} >, f^{-1}(< Y, \{a\}, \varphi >) =< \mathcal{H}, \{a\}, \varphi >$. Then f is strongly $I\alpha_g^{\wedge}$-continuous.

Theorem 3.3: If a map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is strongly $I\alpha_g^{\wedge}$-continuous then it is I-continuous.

Proof: Let P be a I-open set in $(Y, I\tau_\theta)$. Since every I-open set is $I\alpha_g^{\wedge}$-open, P is $I\alpha_g^{\wedge}$-open in $(Y, I\tau_\theta)$. Since f is strongly $I\alpha_g^{\wedge}$-continuous, $f^{-1}(P)$ is I-open in $(\mathcal{H}, I\tau_\mu)$. Therefore, f is I-continuous.

Remark 3.4: The converse of the above theorem is not true as seen from the following example.

Example 3.5: Let $\mathcal{H} = \{a, b\} = Y$, and $I\tau_\mu = \{\mathcal{H}, \varphi, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4\} = I\alpha_g^{\wedge} - O(\mathcal{H})$ where $\mathcal{A}_1 =<$

$\mathcal{H}, \varphi, \{b\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{a\}, \{b\} >, \mathcal{A}_3 = < \mathcal{H}, \{a\}, \varphi >, \mathcal{A}_4 =< \mathcal{H}, \varphi, \varphi >$. $I\tau_\theta = \{Y, \phi, < Y, \varphi, \varphi >, < Y, \{b\}, \varphi >\}$. Then $I\alpha_g^\wedge$-O(Y) $= \{Y, \phi, < Y, \varphi, \varphi >, < Y, \{b\}, \varphi >, < Y, \{a\}, \varphi >, < Y, \varphi, \{a\} >, < Y, \{b\}, \{a\} >\}$. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as $f(a) = b, f(b) = a$. Then $f^{-1}(< Y, \varphi, \varphi >) =< \mathcal{H}, \varphi, \varphi >, f^{-1}(< Y, \{b\}, \varphi >) =< \mathcal{H}, \{a\}, \varphi >$ Then f is I-continuous. But, $f^{-1}(< Y, \{a\}, \varphi >) =< \mathcal{H}, \{b\}, \varphi >$ which is not I-open in $\mathcal{H}$. Hence, f is not strongly $I\alpha_g^\wedge$-continuous.

Theorem 3.6: A map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is strongly $I\alpha_g^\wedge$-continuous if and only if the inverse image of every $I\alpha_g^\wedge$-closed set in $(Y, I\tau_\theta)$ is I-closed in $(\mathcal{H}, I\tau_\mu)$.

Proof: Assume that f is strongly $I\alpha_g^\wedge$-continuous. Let O be any $I\alpha_g^\wedge$-closed set in $(Y, I\tau_\theta)$. Then $Y - O$ is $I\alpha_g^\wedge$-open in $(Y, I\tau_\theta)$. Since f is strongly $I\alpha_g^\wedge$-continuous, $f^{-1}(Y - O)$ is I-open in $(\mathcal{H}, I\tau_\mu)$. But $f^{-1}(Y - O) = \mathcal{H} / f^{-1}(O)$and so $f^{-1}(O)$is I-closed in $(\mathcal{H}, I\tau_\mu)$.

Conversely, assume that the inverse image of every $I\alpha_g^\wedge$-closed set in $(Y, I\tau_\theta)$ is I-closed in $(\mathcal{H}, I\tau_\mu)$. Let $Y - O$ is $I\alpha_g^\wedge$-closed in $(Y, I\tau_\theta)$. By assumption, $f^{-1}(Y - O)$ is I-closed in $(\mathcal{H}, I\tau_\mu)$, but $f^{-1}(Y - O)$ = $\mathcal{H}/f^{-1}(O)$ and so $f^{-1}(O)$ is I-open in $(\mathcal{H}, I\tau_\mu)$. Therefore, f is strongly $I\alpha_g^\wedge$-continuous.

Theorem 3.7: Every strongly I-continuous is strongly $I\alpha_g^\wedge$-continuous.

Proof: Let $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is strongly I-continuous. Let K be any $I\alpha_g^\wedge$-open set in Y. Since f is strongly I-continuous, the inverse image of every subset of Y is both I-open and I-closed in $\mathcal{H}$. Hence $f^{-1}(K)$is I-open in $\mathcal{H}$. Therefore, f is strongly $I\alpha_g^\wedge$-continuous.

Remark 3.8: The converse of the above theorem need not be true as shown in the following example.

Example 3.9: Let $\mathcal{H} = \{a, b\}$ and family $I\tau_\mu = \{\mathcal{H}, \varphi, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4\}$ where $\mathcal{A}_1 =< \mathcal{H}, \varphi, \{b\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{a\}, \{b\} >, \mathcal{A}_3 = < \mathcal{H}, \{a\}, \varphi >, \mathcal{A}_4 =< \mathcal{H}, \varphi, \varphi >$. $Y = \{a, b\}$ with $I\tau_\theta$=$\{Y, \phi, < Y, \varphi, \{b\} >, < Y, \{a\}, \varphi >\}$. Then $I\alpha_g^\wedge$-O(Y) $= \{Y, \phi, < Y, \{a\}, \varphi >, < Y, \varphi, \{b\} >, < Y, \{a\}, \{b\} >\}$. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as $f(a) = a, f(b) = b$. Then, f is strongly $I\alpha_g^\wedge$-continuous. Then $f^{-1}(< Y, \varphi, \{b\} >) =< \mathcal{H}, \varphi, \{b\} >, f^{-1}(< Y, \{a\}, \varphi >) =< \mathcal{H}, \{a\}, \varphi >$ and $f^{-1}(< Y, \{a\}, \{b\} >) =< \mathcal{H}, \{a\}, \{b\} >$. But, $f^{-1}(< Y, \varphi, \{b\} >) =< \mathcal{H}, \varphi, \{b\} >$ which is not I-closed in $\mathcal{H}$. Hence, f is not strongly I-continuous.

Theorem 3.10: Every strongly $I\alpha_g^\wedge$-continuous is $I\alpha_g^\wedge$-continuous.

Proof: Let $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ be strongly $I\alpha_g^\wedge$-continuous. Let L be a I-open set in Y. Then L is $I\alpha_g^\wedge$-open in Y. Since f is strongly

$I\alpha_g^\wedge$-continuous, $f^{-1}(L)$ is I-open in $\mathcal{H}$. Since every I-open is $I\alpha_g^\wedge$-open, $f^{-1}(L)$ is $I\alpha_g^\wedge$-open in $\mathcal{H}$. Therefore, f is $I\alpha_g^\wedge$-continuous.

Remark 3.11: The converse of the above theorem need not be true as shown in the following example.

Example 3.12: Let $\mathcal{H} = \{a, b\}, I\tau_\mu = \{\mathcal{H}, \phi, <\mathcal{H}, \varphi, \varphi>, <\mathcal{H}, \{b\}, \varphi>\}$. Then $I\alpha_g^\wedge$-O($\mathcal{H}$) $= \{\mathcal{H}, \phi, <\mathcal{H}, \varphi, \varphi>, <\mathcal{H}, \{b\}, \varphi>, <\mathcal{H}, \{a\}, \varphi>, <\mathcal{H}, \varphi, \{a\}>, <\mathcal{H}, \{b\}, \{a\}>\}$.

Y={a,b} with IT $I\tau_\theta$=$\{Y, \phi, <Y, \{b\}, \{a\}>, <Y, \varphi, \{a\}>\}$. Then $I\alpha_g^\wedge$-O(Y) $= \{Y, \phi, <Y, \varphi, \varphi>, <Y, \{b\}, \varphi>, <Y, \varphi, \{a\}>, <Y, \varphi, \{b\}>, <Y, \{b\}, \{a\}>\}$. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as $f(a) = a, f(b) = b$. $f^{-1}(<Y, \varphi, \{a\}>) = <\mathcal{H}, \varphi, \{a\}>, f^{-1}(<Y, \{b\}, \{a\}>) = <\mathcal{H}, \{b\}, \{a\}>$. Then f is $I\alpha_g^\wedge$-continuous. But, $f^{-1}(<Y, \varphi, \{a\}>) = <\mathcal{H}, \varphi, \{a\}>$ which is not I-open in $\mathcal{H}$. Hence, f is not strongly $I\alpha_g^\wedge$-continuous.

Theorem 3.13: If a map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is strongly $I\alpha_g^\wedge$-continuous and a map $g: (Y, I\tau_\theta) \rightarrow (Z, I\tau_\rho)$ is $I\alpha_g^\wedge$-continuous then $g \circ f: (\mathcal{H}, I\tau_\mu \rightarrow (Z, I\tau_\rho)$ is I-continuous.

Proof: Let O be any I-open set in Z. Since g is $I\alpha_g^\wedge$-continuous, $g^{-1}(O)$is $I\alpha_g^\wedge$-open in Y.

Since f is strongly $I\alpha_g^\wedge$-continuous $f^{-1}(g^{-1}(O))$ is I-open in $\mathcal{H}$. But,$(g \circ f)^{-1}(O)$ = $f^{-1}(g^{-1}(O))$. Therefore, $g \circ f$ is I-continuous.

Theorem 3.14: If a map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is strongly $I\alpha_g^\wedge$-continuous and a map $g: (Y, I\tau_\theta) \rightarrow (Z, I\tau_\rho)$ is $I\alpha_g^\wedge$-irresolute , then $g \circ f: (\mathcal{H}, I\tau_\mu) \rightarrow (Z, I\tau_\rho)$ is strongly $I\alpha_g^\wedge$-continuous.

Proof: Let S be any $I\alpha_g^\wedge$-open set in Z. Since g is $I\alpha_g^\wedge$-irresolute, $g^{-1}(S)$ is $I\alpha_g^\wedge$-open in Y. Also, f is strongly $I\alpha_g^\wedge$-continuous $f^{-1}(g^{-1}(S))$ is I-open in $\mathcal{H}$. But, $(g \circ f)^{-1}(S)$= $f^{-1}g^{-1}(S)$ is I-open in $\mathcal{H}$. Hence, $g \circ f: (\mathcal{H}, I\tau_\mu) \rightarrow (Z, I\tau_\rho)$ is strongly $I\alpha_g^\wedge$-continuous.

Theorem 3.15: If a map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is $I\alpha_g^\wedge$-continuous and a map $g: (Y, I\tau_\theta) \rightarrow (Z, I\tau_\rho)$ is strongly $I\alpha_g^\wedge$-continuous , then $g \circ f: (\mathcal{H}, I\tau_\mu) \rightarrow (Z, I\tau_\rho)$ is $I\alpha_g^\wedge$-irresolute.

Proof: Let P be any $I\alpha_g^\wedge$-open set in Z. Since g is strongly $I\alpha_g^\wedge$-continuous, $g^{-1}(P)$ is I-open in Y. Also, f is $I\alpha_g^\wedge$-continuous, $f^{-1}(g^{-1}(P))$ is $I\alpha_g^\wedge$-open in $\mathcal{H}$. But $(g \circ f)^{-1}(P) = f^{-1}(g^{-1}(P))$. Hence, $g \circ f: (\mathcal{H}, I\tau_\mu) \rightarrow (Z, I\tau_\rho)$ is $I\alpha_g^\wedge$-irresolute.

Theorem 3.16: Let $(\mathcal{H}, I\tau_\mu)$ be a ITS and $(Y, I\tau_\theta)$ be a $I\alpha_g^\wedge$-$T_{1/2}$ space and $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ be a map. Then the following are equivalent

(1) f is strongly $I\alpha_g^\wedge$-continuous

(2) f is I-continuous

Proof: (1) ⇒(2) Let M be a I-open set in Y. Since every I-open set is $I\alpha_g^{\wedge}$-open, M is $I\alpha_g^{\wedge}$-open in Y. Then $f^{-1}(M)$ is I-open in $\mathcal{H}$. Hence, f is I-continuous. (2) ⇒(1) Let L be any $I\alpha_g^{\wedge}$-open in $(Y, I\tau_\theta)$. Since, $(Y, I\tau_\theta)$ is a $I\alpha_g^{\wedge}$-$T_{1/2}$ space, L is I-open in $(Y, I\tau_\theta)$. Since, f is I-continuous, $f^{-1}(L)$ is I-op
en in $(\mathcal{H}, I\tau_\mu)$. Hence, f is strongly $I\alpha_g^{\wedge}$-continuous.

Theorem 3.17: Let $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ be a map. Both $(\mathcal{H}, I\tau_\mu)$ and $(Y, I\tau_\theta)$ are $I\alpha_g^{\wedge}$-$T_{1/2}$ space. Then the following are equivalent.

(1) f is $I\alpha_g^{\wedge}$-irresolute
(2) f is strongly $I\alpha_g^{\wedge}$-continuous
(3) f is I-continuous
(4) f is $I\alpha_g^{\wedge}$-continuous

Proof: The proof is obvious.

Theorem 3.18: The composition of two strongly $I\alpha_g^{\wedge}$-continuous maps is strongly $I\alpha_g^{\wedge}$- continuous.

Proof: Let O be a $I\alpha_g^{\wedge}$-open set in $(Z, I\tau_\rho)$. Since, g is strongly $I\alpha_g^{\wedge}$-continuous, $g^{-1}(O)$ is I-open in $(Y, I\tau_\theta)$. Then, $g^{-1}(O)$ is $I\alpha_g^{\wedge}$-open in $(Y, I\tau_\theta)$. Also, $f$is strongly $I\alpha_g^{\wedge}$-continuous which implies $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is I-open in $(\mathcal{H}, I\tau_\mu)$. Hence, $(g \circ f)$ is strongly $I\alpha_g^{\wedge}$-continuous.

Theorem 3.19: If $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ and $g : (Y, I\tau_\theta) \rightarrow (Z, I\tau_\rho)$ be any two maps .Then their composition $g \circ f : (\mathcal{H}, I\tau_\mu) \rightarrow (Z, I\tau_\rho)$ is strongly $I\alpha_g^{\wedge}$-continuous if g is strongly $I\alpha_g^{\wedge}$-continuous and f is I-continuous.

Proof: Let S be a $I\alpha_g^{\wedge}$-open in $(Z, I\tau_\rho)$. Since, g is strongly $I\alpha_g^{\wedge}$-continuous, $g^{-1}(S)$ is I-open in $(Y, I\tau_\theta)$. Since f is I-continuous, $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is I-open in $(\mathcal{H}, I\tau_\mu)$. Hence, $(g \circ f)$ is strongly $I\alpha_g^{\wedge}$-continuous.

3. Perfectly $I\alpha_g^{\wedge}$-Continuous Function

Under this section we introduce the concepts of perfectly $I\alpha_g^{\wedge}$-continuous function and we investigate relationships among them and give examples.

Definition 3.1: A map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is said to be **perfectly $I\alpha_g^{\wedge}$-continuous function** if the inverse image of every $I\alpha_g^{\wedge}$-open set in $(Y, I\tau_\theta)$ is both I-open and I-closed in $(\mathcal{H}, I\tau_\mu)$.

Example 3.2: Let $\mathcal{H} = \{a, b\}, I\tau_\mu$ = $\{\mathcal{H}, \phi, <\mathcal{H}, \varphi, \varphi>, <\mathcal{H}, \{a\}, \varphi>, <\mathcal{H}, \varphi, \{a\}>\} = I\tau_\mu{}^c$. Y={a,b} with IT $I\tau_\theta$={Y, $\phi, <Y, \{a\}, \varphi>, <Y, \varphi, \{b\}>$}. Then $I\alpha_g^{\wedge}$-O(Y)= {Y, $\phi, <Y, \{a\}, \varphi>, <Y, \varphi, \{b\}>, <Y, \{a\}, \{b\}>\}$ Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as

$f(a) = a = f(b)$. Then $f^{-1}(\ < Y, \{a\}, \varphi >) =< \mathcal{H}, \mathcal{H}, \varphi >$, $f^{-1}(< Y, \varphi, \{b\} >) =< \mathcal{H}, \varphi, \varphi >$ and $f^{-1}(< Y, \{a\}, \{b\} >) =< \mathcal{H}, \mathcal{H}, \varphi >$ Then f is perfectly $I\alpha_g^{\wedge}$-continuous.

Theorem 3.3: If a map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is perfectly $I\alpha_g^{\wedge}$-continuous then it is strongly $I\alpha_g^{\wedge}$-continuous.

Proof: Assume that f is perfectly $I\alpha_g^{\wedge}$-continuous. Let J be any $I\alpha_g^{\wedge}$-open set in $(Y, I\tau_\theta)$. Since, f is perfectly $I\alpha_g^{\wedge}$-continuous, $f^{-1}(J)$ is I-open in $(\mathcal{H}, I\tau_\mu)$. Therefore, f is strongly $I\alpha_g^{\wedge}$-continuous.

Remark 3.4: The converse of the above theorem need not be true as shown in the following example.

Example 3.5: Let $\mathcal{H} = \{a, b\}$ and family $I\tau_\mu = \{\mathcal{H}, \varphi, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4\}\ where\ \mathcal{A}_1 =< \mathcal{H}, \varphi, \{b\} >$, $\mathcal{A}_2 =< \mathcal{H}, \{a\}, \{b\} >, \mathcal{A}_3 = < \mathcal{H}, \{a\}, \varphi >, \mathcal{A}_4 =< \mathcal{H}, \ \varphi, \varphi >$ $Y = \{a, b\}$ with IT $I\tau_\theta$={Y, $\phi, < Y, \varphi, \{b\} >, < Y, \{a\}, \varphi >\}$. Then $I\alpha_g^{\wedge}$-O(Y) = $\{Y, \phi, < Y, \{a\}, \varphi >, < Y, \varphi, \{b\} >, < Y, \{a\}, \{b\} >\}$. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as $f(a) = a, f(b) = b$. Then $f^{-1}(< Y, \varphi, \{b\} >) =< \mathcal{H}, \varphi, \{b\} >$, $f^{-1}(< Y, \{a\}, \varphi >) =< \mathcal{H}, \{a\}, \varphi >$ and $f^{-1}(< Y, \ \{a\}, \{b\} >) =< \mathcal{H}, \{a\}, \{b\} >$. Then, f is strongly $I\alpha_g^{\wedge}$-continuous. But, $f^{-1}(< Y, \ \varphi, \{b\} >) =< \mathcal{H}, \varphi, \{b\} >$ which is not I-closed in $\mathcal{H}$. Hence, f is not perfectly $I\alpha_g^{\wedge}$-continuous.

Theorem 3.6: If a map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is perfectly $I\alpha_g^{\wedge}$-continuous, then f is perfectly I-continuous.

Proof: Let S be a I-open set in Y. Then S is a $I\alpha_g^{\wedge}$-open set in $(Y, I\tau_\theta)$. Since f is perfectly $I\alpha_g^{\wedge}$-continuous, $f^{-1}(S)$ is both I-open and I-closed in $(\mathcal{H}, I\tau_\mu)$. Therefore, f is perfectly I-continuous.

Remark 3.7 : The converse of the above theorem need not be true as shown in the following example.

Example 3.8: Let $\mathcal{H} = \{a, b\}$ and family $I\tau_\mu = \{\mathcal{H}, \varphi, < \mathcal{H}, \varphi, \{a\} >, < \mathcal{H}, \{a\}, \varphi >, < \mathcal{H}, \varphi, \varphi >\} = I\tau^c$ a $Y = \{a, b\}$ with $I\tau_\theta$={Y, $\phi, < Y, \{b\}, \varphi >, < Y, \varphi, \varphi >\}$. Then $I\alpha_g^{\wedge}$-O(Y) = $\{Y, \phi, < Y, \varphi, \varphi >, < Y, \{b\}, \varphi >, < Y, \varphi, \{a\} >, < Y, \{a\}, \varphi >, < Y, \{b\}, \{a\} >\}$. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as $f(a) = b, f(b) = a$. Then $f^{-1}(< Y, \{b\}, \ \varphi >) =< \mathcal{H}, \{a\}, \varphi >$, $f^{-1}(< Y, \varphi, \varphi >) =< \mathcal{H}, \varphi, \varphi >$. Then f is perfectly I-continuous. Here, $f^{-1}(< Y, \{a\}, \varphi >) =< \mathcal{H}, \{b\}, \varphi >$, which is not I-open and not I-closed set in $(\mathcal{H}, I\tau_\mu)$. Therefore, f is not perfectly $I\alpha_g^{\wedge}$-continuous.

Theorem 3.9: A map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is perfectly $I\alpha_g^{\wedge}$-continuous if and only if $f^{-1}(\mathcal{A})$ is both I-open and I-closed in $(\mathcal{H}, I\tau_\mu)$ for every $I\alpha_g^{\wedge}$-closed set $\mathcal{A}$ in $(Y, I\tau_\theta)$.

Proof: Let $\mathcal{A}$ be any $I\alpha_g^{\wedge}$-closed set in $(Y, I\tau_\theta)$. Then $\mathcal{A}^c$is $I\alpha_g^{\wedge}$-open in $(Y, I\tau_\theta)$. Since, f is perfectly $I\alpha_g^{\wedge}$-continuous, $f^{-1}(\mathcal{A}^c)$ is both I-open and I-closed in $(\mathcal{H}, I\tau_{\mu})$. But $f^{-1}(\mathcal{A}^{c})= \mathcal{H}/f^{-1}(\mathcal{A})$ and so, $f^{-1}(\mathcal{A})$is both I-open and I-closed in $(\mathcal{H}, I\tau_{\mu})$. Conversely, assume that the inverse image of every $I\alpha_{g}^{\wedge}$-closed set in $(Y, I\tau_{\theta})$ is both I-open and I-closed in $(Y, I\tau_{\theta})$. Let $\mathcal{A}$ be any $I\alpha_{g}^{\wedge}$-open in $(Y, I\tau_{\theta})$. Then, $\mathcal{A}^{c}$ is $I\alpha_{g}^{\wedge}$-closed in $(Y, I\tau_{\theta})$. By assumption, $f^{-1}(\mathcal{A}^{c})$ is both I-open and I-closed in $(\mathcal{H}, I\tau_{\mu})$. But, $f^{-1}(\mathcal{A}^{c})$= $\mathcal{H}/f^{-1}(\mathcal{A})$and so, $f^{-1}(\mathcal{A})$ is both I-open and I-closed in $(Y, I\tau_{\theta})$. Therefore, f is perfectly $I\alpha_{g}^{\wedge}$-continuous.

Theorem 3.10: Let $(\mathcal{H}, I\tau_{\mu})$ be a I-discrete topological space and $(Y, I\tau_{\theta})$ be any ITS's. Let $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (Y, I\tau_{\theta})$ be a map, then the following statements are true.

(1) f is strongly $I\alpha_{g}^{\wedge}$-continuous

(2) f is perfectly$I\alpha_{g}^{\wedge}$-continuous

Proof: (1) ⇒(2) Let J be any $I\alpha_{g}^{\wedge}$-open set in $(Y, I\tau_{\theta})$. By hypothesis, $f^{-1}(J)$is I-open in $(\mathcal{H}, I\tau_{\mu})$. Since $(\mathcal{H}, I\tau_{\mu})$ is a I-discrete space, $f^{-1}(J)$is I-closed in $(\mathcal{H}, I\tau_{\mu})$.Then, $f^{-1}(J)$ is both I-open and I-closed in $(\mathcal{H}, I\tau_{\mu})$. Hence, f is perfectly $I\alpha_{g}^{\wedge}$-continuous.

(2) ⇒(1) Let J be any $I\alpha_{g}^{\wedge}$-open set in $(Y, I\tau_{\theta})$. Then, $f^{-1}(J)$ is both I-open and I-closed in $(\mathcal{H}, I\tau_{\mu})$. Hence, f is strongly $I\alpha_{g}^{\wedge}$-continuous.

Theorem 3.11. If $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (Y, I\tau_{\theta})$ and $g : (Y, I\tau_{\theta}) \rightarrow (Z, I\tau_{\rho})$ are perfectly $I\alpha_{g}^{\wedge}$-continuous then, the composition $g \circ f : (\mathcal{H}, I\tau_{\mu}) \rightarrow (Z, I\tau_{\rho})$ is also perfectly $I\alpha_{g}^{\wedge}$-continuous.

Proof: Let K be a $I\alpha_{g}^{\wedge}$-open set in $(Z, I\tau_{\rho})$. Since, g is perfectly $I\alpha_{g}^{\wedge}$-continuous, $g^{-1}(K)$is both I-open and I-closed in $(Y, I\tau_{\theta})$. Since every I-open set is $I\alpha_{g}^{\wedge}$-open, $g^{-1}(K)$is $I\alpha_{g}^{\wedge}$-open in $(Y, I\tau_{\theta})$. Since f is perfectly $I\alpha_{g}^{\wedge}$-continuous, $f^{-1}(g^{-1}(K))$ is both I-open and I-closed in $(\mathcal{H}, I\tau_{\mu})$. But, $f^{-1}(g^{-1}(K)) =(g \circ f)^{-1}(K)$. Hence, $g \circ f$ is perfectly $I\alpha_{g}^{\wedge}$-continuous.

Theorem 3.12: If $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (Y, I\tau_{\theta})$ and $g : (Y, I\tau_{\theta}) \rightarrow (Z, I\tau_{\rho})$ be any two maps . Then, $g \circ f : (\mathcal{H}, I\tau_{\mu}) \rightarrow (Z, I\tau_{\rho})$ is strongly $I\alpha_{g}^{\wedge}$-continuous if g is perfectly I-continuous and f is I-continuous.

Proof: Let O be any $I\alpha_{g}^{\wedge}$-open set in $(Z, I\tau_{\rho})$. Since g is perfectly $I\alpha_{g}^{\wedge}$-continuous , $g^{-1}(O)$ is I-open and I-closed in $(Y, I\tau_{\theta})$. Since, f is I-continuous. $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is I-open in $(\mathcal{H}, I\tau_{\mu})$. Hence, $g \circ f$ is strongly $I\alpha_{g}^{\wedge}$-continuous.

Theorem 3.13: If a map $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (Y, I\tau_{\theta})$ is perfectly $I\alpha_{g}^{\wedge}$-continuous and a map $g : (Y, I\tau_{\theta}) \rightarrow (Z, I\tau_{\rho})$ is strongly $I\alpha_{g}^{\wedge}$-continuous then, $g \circ f : (\mathcal{H}, I\tau_{\mu}) \rightarrow (Z, I\tau_{\rho})$ is perfectly $I\alpha_{g}^{\wedge}$-continuous.

Proof: Let G be any $I\alpha_g^{\wedge}$-open set in $(Z, I\tau_\rho)$. Since g is strongly $I\alpha_g^{\wedge}$-continuous, $g^{-1}(G)$is I-open in $(Y, I\tau_\theta)$. Then, $g^{-1}(G)$is $I\alpha_g^{\wedge}$-open in $(Y, I\tau_\theta)$. By hypothesis, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is both I-open and I-closed in $(\mathcal{H}, I\tau_\mu)$. Therefore, $g \circ f$ is perfectly $I\alpha_g^{\wedge}$-continuous.

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