Journal of Namibian Studies, 33 S3(2023): 3823-3840 ISSN: 2197-5523 (online)

Fuzzy Control Chart For Analysis Of N-Policy FM/FM/1vacation Queueing System With Server Start-Up And Time-Out

P. Sudhakara Babu[1], K.Satish Kumar[2], K. Chandan[3] ,
Ganapathi Swamy Chintada[4]

[1]Research Scholar, Dept.of Statistics , Acharya Nagarjuna University, Guntur, Andhra Pradesh,
[2]Assistant Professor, Dept. Of Statistics, Central University of Rajasthan, Ajmer Dist. Rajasthan,
[3]Professor (Rtd.), Dept. Of Statistics, Acharya Nagarjuna University, Guntur, Andhra Pradesh.
[4]Associare Professor of Biostatistics, Dept. Of Community Medicine,GSL Medical Coolege,Rajahmundry, Andhra Pradesh.

Corresponding auother
Dr Ganapathi Swamy Chintada
Associare Professor of Biostatistics,
Dept. Of Community Medicine,
GSL Medical Coolege,
Rajahmundry.

Abstract:
This paper we constructs the N-policy FM/FM/1 vacation queueing system with server start-up and time-out by using fuzzy control chart based on Zedeh extension law to transform fuzzy queues into family of crisp queues. Arrival rate, service rate and vacation times will all be made by fuzzy nature. We derive the membership functions of the parameters of the control chart corresponding to the expected system length L and also confirmed the parameters of the control chart for fuzzy. We have also set an numerical example.

Keywords: Time out, N-Policy, Zedeh extension principle, fuzzy queues, controlchart.

1.Introduction

The principle of queues plays a significant part in our daily lives. Everyone must follow the queue in order to provide services to everybody to ensure prompt service for all. According to a given queue discipline, different types of servers serve different sorts of customers.

Vacation queueing system may be either single or multiple. In single vacation policy, immediately the system is free, the server takes exactly one vacation and provides service for the customers if exists any otherwise remains idle.

The term vacation queue was first began in 1970s. Doshi[1] wrote a good survey paper on vacation models.

Time out means taking a break at the time of working hours. Concept of time out was first introduced by Oliver C. Ibe[2] who derived explicit expressions for the average time for waiting on vacation queueing system with server time out.
K. Satish Kumar, K. Chandan et. al [3,4] have worked on this Timeout concepts of queues M/M/1, M^X /G/1 etc.derivedexpressions for the system length with N-policy, vacation, server startup and timeout.

Typically, in queuing theory, it is presupposed that the arrival time and the service time are distributed according to certain probabilities with fixed parameters. But in many real-world scenarios, labels like "fast,""slow," and "medium" may explain arrival patterns and service patterns more or less completely with probability distributions.By supposing the parameters of the system as fuzzy numbers, the queuing theory models thus developed have a extensivescope in real times.The fuzzy queues described by many researchers J.J. Buckely,Chen and many others [5,6,14] studied performance measures of various queueing systems with such fuzzy nature.
The control chart was originated in 1920s by Walter A. Shewhart of Bell Labs which gives a crisp idea of the state of a process by plotting the values of a variable under study on a chart bounded by upper and lower control limits. Montgomery [7] proposed a number of applications of Shewhart control charts in quality analysis in the manufacturing industries. T.poongodi and S. Muthulakshmi[13] constructed and studied the control chart for M/M/S queueing modelwaiting time. A. Pandurangan & R. Varadharajan[8] analysed α-cut fuzzycontrol charts $\overline{\overline{\tilde{X}}}$ - $\tilde{R}$ and $\overline{\overline{\tilde{X}}}$ - $\tilde{S}$ based on trapezoidal fuzzy number. T.Poongodi, Dr.(Mrs)S.Muthulaksmi [9,10] analysed Fuzzy control chart for triangularfuzzy queueing model of Ek/M/1 and also derived control chart for fuzzy triangular number of mean waiting time in M/M/S queueing model. Mohammed Shapique.A [11] developed the performance measures of fuzzy queueing system in waiting time for trapezoidal fuzzy numbers using fuzzy control charts.
We aimed to study on N-policy FM/FM/1 vacation queueing system with server startup and timeout using fuzzy control chart in this paper. Mysterious membership degrees built on Zadeh's [12] expansion law and we also given numerical examples.

2.Model Description

To apply Fuzzy condition, we have taken the expression of expected system length of N-policy FM/FM/1 vacation queueing system with server start-up and timeout taken from reference no [3]:

$$E(L) = \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

The variance of system length is derived as

$$Var(L) = \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right]\frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)} - (E(L))^2$$

And optimal strategy

$$N^* = \sqrt{\left(\frac{2\lambda\mu\gamma+2\lambda\mu c+c\lambda\gamma}{2c\mu\gamma}\right)^2 - \frac{2\lambda^2\mu-2\lambda\gamma\mu+2\lambda^2\gamma-\lambda\mu c+c^2\lambda}{c\mu\gamma} + \frac{2(\mu-\lambda)\left\{c_b\frac{\lambda}{\mu}+c_v\left(\frac{\lambda}{c}+\frac{\lambda}{\gamma}\right)+c_m\frac{\lambda}{\gamma}+c_t\frac{\lambda}{c}+c_s\lambda\right\}}{\mu c_h}} - \frac{2\lambda\mu\gamma+2\lambda\mu c+c\lambda\gamma}{2c\mu\gamma}$$

The upper and lower boundaries (control limits) for the expected system length are given by

$$UCL = E(L) + 3\sqrt{var(L)}$$

$$CL = E(L)$$

$$LCL = E(L) - 3\sqrt{var(L)}$$

3.Model with fuzzy parameter

We considered a single server queueing system which the customer arrives at a fuzzy arrival rate $\tilde{\lambda}$, fuzzy service rate $\tilde{\mu}$and vacation time $\tilde{\gamma}$, $\widetilde{N}$ are approximately known. We construct the membership functions of the fuzzy control chart for expected system length of FM/FM/1 queueing system with server start-up and time-out and are given as follows

$\tilde{\lambda} = \{(x,\eta_{\tilde{\lambda}}(x))/x\in X\}$

$\tilde{\mu} = \{(y,\eta_{\tilde{\mu}}(y))/y\in Y\}$

$\tilde{\gamma} = \{(z,\eta_{\tilde{\gamma}}(z))/z\in Z\}$

$\widetilde{N} = \{(z_1,\eta_{\widetilde{N}}(z_1))/z_1\in Z_1\}$

$\eta_{\tilde{\lambda}}(x)$,$\eta_{\tilde{\mu}}(y)$ and $\eta_{\tilde{\gamma}}(z)$,$\eta_{\widetilde{N}}(z_1)$ are membership functions of triangular and trapezoidal fuzzy numbers. X,Y and Z ,Z_1 are the help of the fuzzy number $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{\gamma}$,$\widetilde{N}$respectively.

Let P(x,y,z,z_1) and $\widetilde{P}$ ($\tilde{\lambda}$, $\tilde{\mu}$, $\tilde{\gamma}$, $\widetilde{N}$) represent the control chart parameter in crisp and fuzzy environment of expected system length L respectively.WhereP refers to the control standards of CL,UCL and LCL. If $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{\gamma}$, $\widetilde{N}$ are fuzzy parameters,then$\widetilde{P}$ ($\tilde{\lambda}$, $\tilde{\mu}$, $\tilde{\gamma}$, $\widetilde{N}$) is also fuzzy. By applying Zadeh's extension law thecontrol chart membership functions forthe expected system length $\widetilde{P}$ ($\tilde{\lambda}$, $\tilde{\mu}$, $\tilde{\gamma}$, $\widetilde{N}$) is defined as

$$\eta_{p(\tilde{\lambda},\tilde{\mu},\tilde{\gamma},\widetilde{N})}(\omega)= \sup_{x\epsilon X, y\epsilon Y, z\epsilon Z, z1\epsilon Z1} min\{ \eta_{\tilde{\lambda}}(x), \eta_{\tilde{\mu}}(y), \eta_{\tilde{\gamma}}(z), \eta_{\widetilde{N}}(z_1)\} \quad (1)$$

4.Control chart parameters for the expected system length of the customers

The fuzzy control limits for expected system length L is given by

$$CL(x,y,z)^L = \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

$$Var(x,y,z)^L = \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)} - (CL(x,y,z))^2$$

$UCL(x,y,z)^L = CL(x,y,z) + 3\sqrt{var(x,y,z)}$ and

$LCL(x,y,z)^L = CL(x,y,z) - 3\sqrt{var(x,y,z)}$

A mathematical procedureis developed forLCL,UCL and CLto obtain the desired membership functions basedon α-cuts.

4.1.The α-cuts approach depend on the extension law

The α-cuts of$\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{\gamma}$, $\widetilde{N}$are crisp intervals and is defined as

$\lambda_\alpha = \{x\epsilon X / \eta_{\tilde{\lambda}}(x) \geq \alpha\}$
(2)

$\mu_\alpha = \{(y\epsilon Y / \eta_{\tilde{\mu}}(y) \geq \alpha\}$
(3)

$\gamma_\alpha = \{(z\epsilon Z / \eta_{\tilde{\gamma}}(z) \geq \alpha\}$
(4)

$N_\alpha = \{(z_1\epsilon Z_1 / \eta_{\widetilde{N}}(z_1) \geq \alpha)\}$
(5)

These crisp sets can be Expressed in this way

$\lambda_\alpha = [x_\alpha^L, x_\alpha^U] = [\min_{x \epsilon X}\{x\epsilon X / \eta_{\tilde{\lambda}}(x) \geq \alpha\}, \{\max_{x \epsilon X} x\epsilon X / \eta_{\tilde{\lambda}}(x) \geq \alpha\}]$
(6)

$\mu_\alpha = [y_\alpha^L, y_\alpha^U] = [\min_{y \epsilon Y}\{ y\epsilon Y / \eta_{\tilde{\mu}}(y) \geq \alpha\}, \{\max_{y \epsilon Y} y\epsilon Y / \eta_{\tilde{\mu}}(Y) \geq \alpha\}]$ (7)

$\gamma_\alpha = [z_\alpha^L, z_\alpha^U] = [\min_{z \epsilon Z}\{ z\epsilon Z / \eta_{\tilde{\gamma}}(z) \geq \alpha\}, \{\max_{z \epsilon Z} z\epsilon Z / \eta_{\tilde{\gamma}}(z) \geq \alpha\}]$ (8)

$N_\alpha = [z_{1\alpha}^L, z_{1\alpha}^U] = [\min_{z1 \epsilon Z1}\{ z1\epsilon Z1 / \eta_{\tilde{\gamma}}(z_1) \geq \alpha\}, \{\max_{z1 \epsilon Z1} z1\epsilon Z1 / \eta_{\tilde{\gamma}}(z) \geq \alpha\}]$ (9)

The above intervals provide information on the arrival rate, the service and vacation time rate with possibility α.

The bounds of above intervals in (6), (7) and (8),(9) are basis of α and can be obtained as

$x_\alpha^L = \min \eta_{\tilde{\lambda}}^{-1}(\alpha)$, $x_\alpha^U = \max \eta_{\tilde{\lambda}}^{-1}(\alpha)$,

$y_\alpha^L = \min \eta_{\tilde{\mu}}^{-1}(\alpha)$, $y_\alpha^U = \max \eta_{\tilde{\mu}}^{-1}(\alpha)$

$z_\alpha^L = \min \eta_{\tilde{\gamma}}^{-1}(\alpha)$, $z_\alpha^U = \max \eta_{\tilde{\gamma}}^{-1}(\alpha)$

$Z_{1\alpha}^L = \min \eta_{\tilde{N}}^{-1}(\alpha)$, $N_\alpha^U = \max \eta_{\tilde{N}}^{-1}(\alpha)$

Hence, the membership function can be built by using α-cuts.

4.2.Building of membership function

We consider the membership functions of the limitations of the control chart for expected system length L. In equation (1), $\eta_{\widetilde{CL}}(\omega)$ is the minimum standard of $\eta_{\tilde{\lambda}}(x)$,$\eta_{\tilde{\mu}}(y)$ and $\eta_{\tilde{\gamma}}(z)$, $\eta_{\gamma}(z_1)$ to allocate with the help of the membership function, we require to hold such that ω = CL(x,y,z) and $\eta_{\widetilde{CL}}(\omega) = \alpha$ at least one of the following four cases

(i) :$\eta_{\tilde{\lambda}}(x) = \alpha$, $\eta_{\tilde{\mu}}(y) \geq \alpha$, $\eta_{\tilde{\gamma}}(z) \geq \alpha$, $\eta_{\gamma}(z_1) \geq \alpha$
(ii) : $\eta_{\tilde{\lambda}}(x) \geq \alpha$, $\eta_{\tilde{\mu}}(y) = \alpha$, $\eta_{\tilde{\gamma}}(z) \geq \alpha$, $\eta_{\gamma}(z_1) \geq \alpha$ (8)
(iii) :$\eta_{\tilde{\lambda}}(x) \geq \alpha$, $\eta_{\tilde{\mu}}(y) \geq \alpha$ $\eta_{\tilde{\gamma}}(z) = \alpha$, $\eta_{\gamma}(z_1) \geq \alpha$
(iv) : $\eta_{\tilde{\lambda}}(x) \geq \alpha$, $\eta_{\tilde{\mu}}(y) \geq \alpha$, $\eta_{\tilde{\gamma}}(z) \geq \alpha$, $\eta_{\gamma}(z_1) = \alpha$

From the definition of λ_{α}, $\mu_{\alpha}$and $\gamma_{\alpha, N\alpha}$in equations (2)-(4), x$\in\lambda_{\alpha}$, y$\in$ $\mu_{\alpha}$and z$\in\gamma_{\alpha}$ may be replaced by x $\in[x_{\alpha}^{L}, x_{\alpha}^{U}]$, y$\in[y_{\alpha}^{L}, y_{\alpha}^{U}]$ and z$\in[z_{\alpha}^{L}, z_{\alpha}^{U}]$,$[z_{1\alpha}^{L}, z_{1\alpha}^{U}]$ respectively. The non-linearparametric programs (NLPs)are formulated in thefollowing to find the upper and lowerbounds of the α-cut of $\eta_{\widetilde{CL}}$(ω) corresponding to allfour cases mentioned in (8)

Case (1):

$$(CL)_{\alpha}^{L_1} = \min \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

Subject to $x_{\alpha}^{L} \leq X \leq x_{\alpha}^{U}$, y $\in\mu_{\alpha}$and z $\in\gamma_{\alpha}$, $z_1\in N_{\alpha}$

$$(CL)_{\alpha}^{U_1} = \max \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

Subject to $x_{\alpha}^{L} \leq X \leq x_{\alpha}^{U}$, y $\in\mu_{\alpha}$and z $\in\gamma_{\alpha}$, $z_1\in N_{\alpha}$

case (ii):

$$(CL)_{\alpha}^{L_2} = \min \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

Subject to x $\in\lambda_{\alpha}$,$y_{\alpha}^{L} \leq Y \leq y_{\alpha}^{U}$, y $\in\mu_{\alpha}$and z $\in\gamma_{\alpha}$, $z_1\in N_{\alpha}$

$$(CL)_{\alpha}^{U_2} = \max \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

Subject to x $\in\lambda_{\alpha}$,$y_{\alpha}^{L} \leq Y \leq y_{\alpha}^{U}$, and z $\in\gamma_{\alpha}$, $z_1\in N_{\alpha}$

case (iii):

$$(CL)_{\alpha}^{L_3} = \min \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

Subject to x $\in\lambda_{\alpha}$,, y $\in\mu_{\alpha}$and $z_{\alpha}^{L} \leq z \leq z_{\alpha}^{U}$,$z_1\in N_{\alpha}$

$$(CL)_{\alpha}^{U_3} = \max \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

Subject to x $\in\lambda_{\alpha}$,, y $\in\mu_{\alpha}$and $z_{\alpha}^{L} \leq Z \leq z_{\alpha}^{U}$, $z_1\in N_{\alpha}$

Case (IV)

$$(CL)_{\alpha}^{L_3} = \min \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

Subject to x $\epsilon\lambda_{\alpha}$,, y $\epsilon\mu_{\alpha}$and z $\epsilon\gamma_{\alpha,}$ $z_{1\alpha}^{L} \le Z_1 \le z_{1\alpha}^{U}$

$$(CL)_{\alpha}^{U_3} = \max \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

Subject to x $\epsilon\lambda_{\alpha}$,, y $\epsilon\mu_{\alpha}$and z $\epsilon\gamma_{\alpha,}$ $z_{1\alpha}^{L} \le Z_1 \le z_{1\alpha}^{U}$

Using the above cases (1),(2)we find the left appear L(ω) and the right appearR(ω) of $\eta_{\widetilde{CL}}$(ω) and we used to notice the lower bound $(CL)_{\alpha}^{L}$ and the upper bound$(CL)_{\alpha}^{U}$ of α-cuts of $\widetilde{CL}$.

These may be rewritten as

$$(CL)_{\alpha}^{L} = \min_{x\epsilon X, y\epsilon Y, z\epsilon Z, z1\epsilon Z1} \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{\mu}{\mu-\lambda} P_0^0$$

Subject to $x_{\alpha}^{L} \le X \le x_{\alpha}^{U}$, $y_{\alpha}^{L} \le Y \le y_{\alpha}^{U}$, $z_{\alpha}^{L} \le Z \le z_{\alpha}^{U}$, $z_{1\alpha}^{L} \le Z_1 \le z_{1\alpha}^{U}$ (9)

$$(CL)_{\alpha}^{U} = \max_{x\epsilon X, y\epsilon Y, z\epsilon Z} \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \left[\frac{N(N-1)}{2} + \frac{\lambda}{\gamma}\left(\frac{\lambda}{\gamma} + N\right)\right] \frac{\mu}{\mu-\lambda} P_0^0$$

Subject to $x_{\alpha}^{L} \le X \le x_{\alpha}^{U}$, $y_{\alpha}^{L} \le Y \le y_{\alpha}^{U}$,$z_{\alpha}^{L} \le Z \le z_{\alpha}^{U}$,$z_{1\alpha}^{L} \le Z_1 \le z_{1\alpha}^{U}$(10)

At least one x, y and z,z_1 must lie within the bound of equations (9), (10) to satisfy the condition $\eta_{\widetilde{CL}}$ (ω) = α. Theseset of mathematical procedures falls into the list of non -linearparametric programs, which facilitateaplannedprocess of how optimal solutions evovle for the ranges of x_{α}^{L}, x_{α}^{U}, y_{α}^{L}, y_{α}^{U}, z_{α}^{L},z_{α}^{U},$z_{1\alpha}^{L}$,$z_{1\alpha}^{U}$andα having the interval [0,1]. The interval [$(CL)_{\alpha}^{L}$, $(CL)_{\alpha}^{U}$] is a crisp interval constitute the α-cuts of ($\widetilde{CL}$). Anew based on the extension law and the convexity possessions of fuzzy numbers we obtained [$(CL)_{\alpha 1}^{L}$, $(CL)_{\alpha 1}^{U}$]⊆[$(CL)_{\alpha 2}^{L}$, $(CL)_{\alpha 2}^{U}$] for $0<\alpha_4<\alpha_3<\alpha_2<\alpha_1<1$. on the other hand, as α increases, the value of $(CL)_{\alpha}^{L}$ increase and $(CL)_{\alpha}^{U}$ decrease. The α- cuts provide a feasible, range of performance measures. A rangeat α=0 is calculated for thehelp of the performance measures andthe all most allfeasible range at α=1is computed for the performance measure. If both the lower and upper bounds$(CL)_{\alpha}^{L}$, $(CL)_{\alpha}^{U}$ of the α- cuts of ($\widetilde{CL}$) are revertible with respect to α, than a left function appearance L(ω) and a right functionappearance R(ω) may be obtained as L(ω) =$[(CL)_{\alpha}^{L}]^{-1}$ and

R(ω)=[($CL)_{\alpha}^{U}$]$^{-1}$. Than the membership function $\eta_{\widetilde{CL}}$ (ω) can be expressed as

$$\eta_{\widetilde{CL}}(\omega) = \begin{cases} L(\omega), & (CL)_{\alpha=0}^{L} \leq \omega \leq (CL)_{\alpha=1}^{L} \\ 1, & (CL)_{\alpha=1}^{L} \leq \omega \leq (CL)_{\alpha=1}^{U} \\ R(\omega), & (CL)_{\alpha=1}^{U} \leq \omega \leq (CL)_{\alpha=0}^{U} \end{cases}$$

Yager index giventhe representative value of the fuzzy number andregionrecompense is used to defuzzify($\widetilde{CL}$) of the expected system length into a crisp one. The Robust ranking index is

$$Y(\widetilde{CL}) = \frac{1}{2}\int_0^1 [(CL)_{\alpha}^{L} + (CL)_{\alpha}^{U}]d\alpha$$

The membership functions $\eta_{\widetilde{UCL}}$(ω) and $\eta_{\widetilde{LCL}}$(ω) and Yager indices can be obtained by following the same procedure for the control chart parameters namely UCL and LCL.

4.3Control chart parameters for expected system length

The standards of the fuzzy control chart for expected length is

$$CL(x,y,z) = \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

$$UCL(x,y,z) = CL(x,y,z) + 3\sqrt{var(x,y,z)}$$

$$LCL(x,y,z) = CL(x,y,z) - 3\sqrt{var(x,y,z)}$$

By following the same method as we used in the previous section, we construct the membership functions and the Yager ranking indices of the parameters of fuzzy control chart relating to the expected system length L.

5.Numerical analysis

5.1.Triangular fuzzy number

Consider an FM/FM/1, Queueing System with Server startup and time out where arrival stream and service stream and idle time aretriangular fuzzy numbers denoted by λ=[1,2,3]; μ = [5,6,7]; γ = [2,3,4], C=1,Cb=300, Cm= 200, Ct=30, Cs=500,Cv=15 and Ch=5.The upper bound and lower bound of $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{\gamma}$ are obtained as

$[x_\alpha^L, x_\alpha^U]$ = [1+α,3-α], $[y_\alpha^L, y_\alpha^U]$ = [5+α,7-α], $[z_\alpha^L, z_\alpha^U]$ = [2+α,4-α]

1. Optimal N-policy(N*)

$$N^* = \sqrt{\left(\frac{2\alpha^3+19\alpha^2+49\alpha+32}{2\alpha^2+14\alpha+20}\right)^2 - \frac{6\alpha^3+38\alpha^2+65\alpha+29}{\alpha^2+7\alpha+10} + \left(8\frac{545\alpha^3+4875\alpha^2+11455\alpha+7125}{\alpha^2+7\alpha+10}\right)/(25+2\alpha)} - \frac{2\lambda\mu\gamma+2\lambda\mu c+c\lambda\gamma}{2c\mu\gamma}$$

Control chart parameter for expected system length L as follows

$$\mathrm{CL}(x,y,z)_\alpha^L = \frac{\alpha^3+11\alpha^2+35\alpha+25}{4\alpha^2+40\alpha+100} + \frac{\alpha^2N^2+N\alpha^2+2\alpha^2+2N^2+3\alpha N^2+2N+3N\alpha+3\alpha+2)}{2(\alpha^2+7\alpha+10)(\alpha^2+N\alpha+4\alpha+2N+3)}$$

$$\mathrm{CL}(x,y,z)_\alpha^U = \frac{3\alpha^2-16\alpha+21}{2\alpha^2-18\alpha+28} + \frac{\alpha^2N^2+N\alpha^2+2\alpha^2+\alpha^2N-12\alpha+12N^2-7\alpha N^2+12N-7N\alpha+18)}{2(\alpha^2-11\alpha+28)(\alpha^2-N\alpha-8\alpha+4N+15)}$$

$$\mathrm{var}(x,y,z)_\alpha^L = \frac{\alpha^3+11\alpha^2+35\alpha+25}{4\alpha^2+40\alpha+100} + \left[\frac{N(N-1)}{2} + \frac{\alpha^2+\alpha^2N+3N\alpha+2N+2\alpha+1}{\alpha^2+4\alpha+4}\right]\frac{\alpha^2N^2+N\alpha^2+2\alpha^2+2N^2+3\alpha N^2+2N+3N\alpha+3\alpha+2)}{2(\alpha^2+7\alpha+10)(\alpha^2+N\alpha+4\alpha+2N+3)} - (\mathrm{CL}(x,y,z))^2$$

$$\mathrm{var}(x,y,z)_\alpha^U = \frac{3\alpha^2-16\alpha+21}{2\alpha^2-18\alpha+28} + \left[\frac{N(N-1)}{2} + \frac{\alpha^2+\alpha^2N-7N\alpha+12N-5\alpha+6}{\alpha^2-8\alpha+16}\right]\frac{\alpha^2N^2+N\alpha^2+2\alpha^2+\alpha^2N-12\alpha+12N^2-7\alpha N^2+12N-7N\alpha+18)}{2(\alpha^2-11\alpha+28)(\alpha^2-N\alpha-8\alpha+4N+15)} - (\mathrm{CL}(x,y,z))^2$$

$$\mathrm{UCL}(x,y,z)_\alpha^L = \mathrm{CL}(x,y,z)_\alpha^L + 3\sqrt{var(x,y,z)}_\alpha^L$$

$$\mathrm{LCL}(x,y,z)_\alpha^L = \mathrm{CL}(x,y,z)_\alpha^L - 3\sqrt{var(x,y,z)}_\alpha^L$$

$$\mathrm{UCL}(x,y,z)_\alpha^U = \mathrm{CL}(x,y,z)_\alpha^L + 3\sqrt{var(x,y,z)}_\alpha^U$$

$$\mathrm{LCL}(x,y,z)_\alpha^U = \mathrm{CL}(x,y,z)_\alpha^L - 3\sqrt{var(x,y,z)}_\alpha^U$$

The values control chart parametersFor different values of α ∈ [0,1] are calculated in table.1

α	x_α^L	x_α^U	y_α^L	y_α^U	z_α^L	z_α^U	N_α^L	N_α^U	CL_α^L

0	1	3	5	7	2	4	13.48840615	17.80681594	0.905258153
0.1	1.1	2.9	5.1	6.9	2.1	3.9	13.96098923	17.73442948	0.966563039
0.2	1.2	2.8	5.2	6.8	2.2	3.8	14.39892226	17.65505171	1.023615857
0.3	1.3	2.7	5.3	6.7	2.3	3.7	14.80666822	17.568138	1.076847711
0.4	1.4	2.6	5.4	6.6	2.4	3.6	15.18782304	17.47308481	1.126647776
0.5	1.5	2.5	5.5	6.5	2.5	3.5	15.54533299	17.369221	1.173365244
0.6	1.6	2.4	5.6	6.4	2.6	3.4	15.88164699	17.25579753	1.217312402
0.7	1.7	2.3	5.7	6.3	2.7	3.3	16.19882626	17.13197493	1.258768092
0.8	1.8	2.2	5.8	6.2	2.8	3.2	16.49862488	16.99680836	1.297981178
0.9	1.9	2.1	5.9	6.1	2.9	3.1	16.78255011	16.84922919	1.33517383
1.0	2	2	6	6	3	3	17.05190842	16.68802249	1.370544523

CL_{α}^{U}	var_{α}^{L}	var_{α}^{U}	UCL_{α}^{L}	UCL_{α}^{U}	LCL_{α}^{L}	LCL_{α}^{U}
1.585970704	59.20220636	134.8056016	23.98814916	36.41771505	-22.17763285	-33.24577364
1.584935172	67.15638953	134.8874483	25.55126368	36.42725189	-23.61813761	-33.25738155
1.583600273	74.95436767	134.8004747	26.99647302	36.41468226	-24.9492413	-33.24748172
1.581966185	82.5481933	134.5301199	28.3336586	36.37810217	-26.17996318	-33.2141698
1.580042113	89.9025656	134.0609485	29.57173681	36.31544965	-27.31844126	-33.15536542
1.577850461	96.99213659	133.3766453	30.71874101	36.22449255	-28.37201052	-33.06879163
1.575432948	103.799398	132.4600278	31.78190927	36.10281701	-29.34728446	-32.95195111
1.572860202	110.3130184	131.293151	32.76776944	35.94782725	-30.25023325	-32.80210684
1.570250788	116.5265307	129.8581175	33.68221771	35.75684257	-31.08625536	-32.616341
1.56782899	122.4372912	128.1422441	34.53058949	35.52780834	-31.86024183	-32.39215036

1.566150499	128.0456531	126.1641612	35.31772228	35.2629968	-32.5766332 3	-32.13069581

From above tables at α = 0 we observe that the value of CL lie in the range [0.9052,1.5859] which implies that CL of E(L) can't exceed 1.5859 or fall before 0.9052 and UCL lies in the range 23.9881 ≤ UCL≤36.4177.

From above tables at α = 1 we observe that the value of CL lie in the range [1.3705,1.5661] which implies that CL of E(L) can't exceed 1.5661 or fall before 1.3705 and UCL lies in the range 35.3177 ≤ UCL≤35.2629.

The expected CL and UCL of E(L) are obtained by applying the Yager ranking index

$R(\widetilde{CL}) = \frac{1}{2}\int_0^1 [(CL)_\alpha^L + (CL)_\alpha^U] d\alpha = 1.24$

$R(U\widetilde{CL}) = \frac{1}{2}\int_0^1 [(UCL)_\alpha^L + (LCL)_\alpha^U] d\alpha = 30.20$

By using MATLAB the inverse function of L(z) and R(z) of $(CL)_\alpha^L$, $(CL)_\alpha^U$, $(UCL)_\alpha^L$&$(UCL)_\alpha^U$ are obtain the membership function of

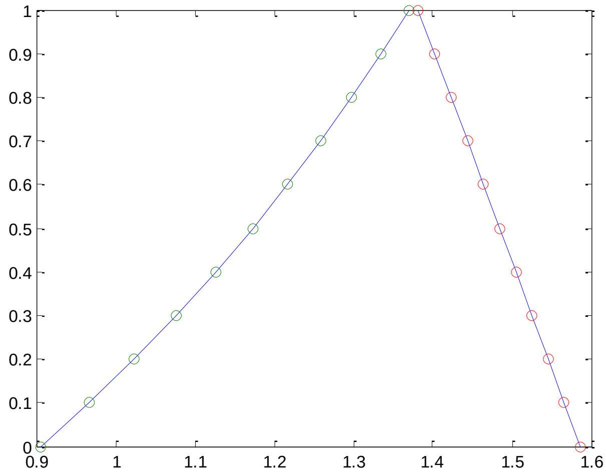
$\eta_{\widetilde{CL}}(\omega)$ and $\eta_{\widetilde{UCL}}(\omega)$ can stated as

$$\eta_{\widetilde{CL}}(\omega) = \begin{cases} L(z), & 0.9052 \leq \omega \leq 1.3705 \\ 1, & 1.3705 \leq \omega \leq 1.5661 \\ R(z), & 1.5661 \leq \omega \leq 1.5859 \end{cases}$$

$$\eta_{\widetilde{UCL}}(\omega) = \begin{cases} L(z), & 23.9881 \leq \omega \leq 35.3177 \\ 1, & 35.2629 \leq \omega \leq 35.3177 \\ R(z), & 35.2629 \leq \omega \leq 36.4177 \end{cases}$$

Figure :1.1 (Triangular CL(L))

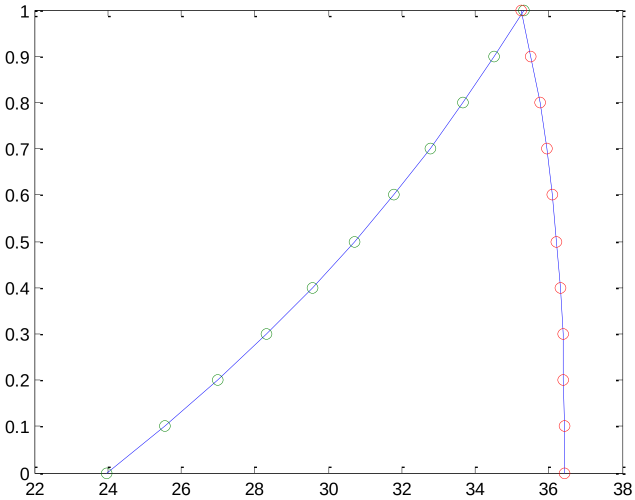
Values of α



Membership function of CL for expected system length L

Figure:1.2 (Triangular UCL (L))

Values of α

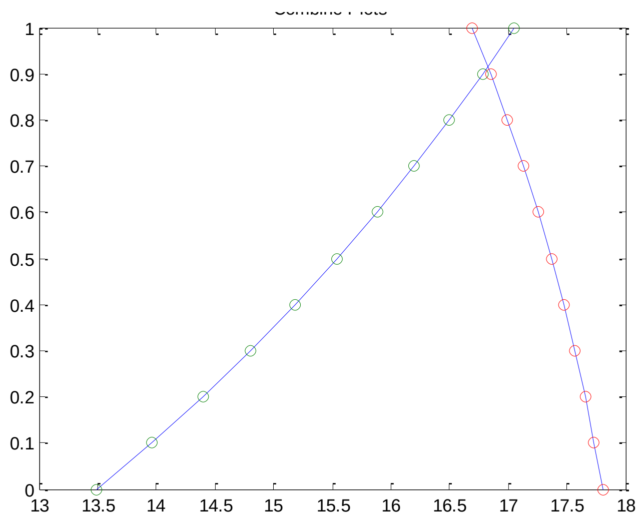


Membership function of UCL for expected system length L

The graph of the membership function of $\eta_{\widetilde{CL}}$ (ω) , $\eta_{\widetilde{UCL}}$ (ω) corresponding to α-cuts relating to E(L) are shown in fig.1.1 and 1.2 respectively. And also given the graphs of the optimum threshold N* and variance (L).

Figure :1.3 (Triangular N)

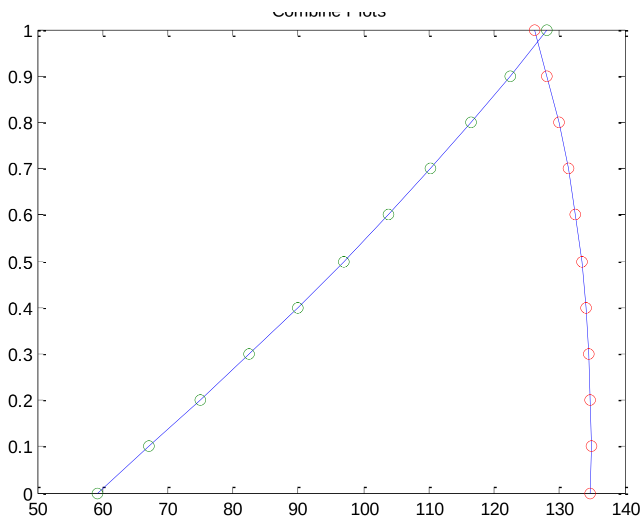
Values of α



Values of N

Figure:1.4(Triangular (var L))

Values of α



Values of var (L)

5.2.Trapezoidal fuzzy numbers

Consider an FM/FM/1, Queueing System with Server startup and time out where arrival pattern and service pattern aretriangular fuzzy numbers denoted by λ=[1,2,3,4]; μ = [5,6,7,8]; γ = [2,3,4,5], C=1, C_b=300,

Cm= 200, Ct=30, Cs=500,Cv=15 and Ch=5. The upper bound and lower bound of $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{\gamma}$ are obtained as
$[x_\alpha^L, x_\alpha^U]$ = [1+α,4-α], $[y_\alpha^L, y_\alpha^U]$ = [5+α,8-α], $[z_\alpha^L, z_\alpha^U]$ = [2+α,5-α]

α	x_α^L	x_α^U	y_α^L	y_α^U	z_α^L	z_α^U	N_α^L	N_α^U	CL_α^L
0	1	3	5	7	2	4	13.48840615	20.79204693	0.905258153
0.1	1.1	2.9	5.1	6.9	2.1	3.9	13.96098923	20.62494041	0.966563039
0.2	1.2	2.8	5.2	6.8	2.2	3.8	14.39892226	20.45101911	1.023615857
0.3	1.3	2.7	5.3	6.7	2.3	3.7	14.80666822	20.269769	1.076847711
0.4	1.4	2.6	5.4	6.6	2.4	3.6	15.18782304	20.08062312	1.126647776
0.5	1.5	2.5	5.5	6.5	2.5	3.5	15.54533299	19.88295408	1.173365244
0.6	1.6	2.4	5.6	6.4	2.6	3.4	15.88164699	19.67606533	1.217312402
0.7	1.7	2.3	5.7	6.3	2.7	3.3	16.19882626	19.45918072	1.258768092
0.8	1.8	2.2	5.8	6.2	2.8	3.2	16.49862488	19.23143198	1.297981178
0.9	1.9	2.1	5.9	6.1	2.9	3.1	16.78255011	18.99184359	1.33517383
1.0	2	2	6	6	3	3	17.05190842	18.73931437	1.370544523

CL_α^U	var_α^L	var_α^U	UCL_α^L	UCL_α^U	LCL_α^L	LCL_α^U
1.888364389	59.20220636	218.1629487	23.98814916	46.1993849	-22.17763285	-42.42265612
1.865487616	67.15638953	214.8591988	25.55126368	45.8397163	-23.61813761	-42.10874107
1.842204418	74.95436767	211.4314328	26.99647302	45.4642504	-24.9492413	-41.77984156
1.818469582	82.5481933	207.8721809	28.3336586	45.07178885	-26.17996318	-41.43484969
1.794233105	89.9025656	204.1733682	29.57173681	44.66100712	-27.31844126	-41.07254091
1.769439611	96.99213659	200.326257	30.71874101	44.23043716	-28.37201052	-40.69155794
1.744027681	103.799398	196.3213797	31.78190927	43.77844711	-29.34728446	-40.29039175

1.71792908	110.3130184	192.1484643	32.7677694 4	43.30321709	- 30.2502332 5	-39.86735893
1.691067859	116.5265307	187.7963486	33.6822177 1	42.80271022	- 31.0862553 6	-39.4205745
1.663359317	122.4372912	183.252882	34.5305894 9	42.27463778	- 31.8602418 3	-38.94791914
1.634708776	128.0456531	178.5048135	35.3177222 8	41.71641684	- 32.5766332 3	-38.44699928

From above tables at α = 0 we observe that the value of CL lie in the range [0.9052,1.8883] which implies that CL of E(L) can't exceed 1.8883 or fall before 0.9052 and UCL lies in the range 23.9881 ≤ UCL≤46.1993. From above tables at α = 1 we observe that the value of CL lie in the range [1.3705,1.6347] which implies that CL of E(L) can't exceed 1.6347 or fall before 1.3705 and UCL lies in the range 35.3177 ≤ UCL≤41.7164

By applying Robust ranking technique the expected CL and UCL of E(L) are

$$Y(\widetilde{CL}) = \frac{1}{2}\int_0^1[(CL)_\alpha^L + (CL)_\alpha^U]d\alpha = 1.39$$

$$Y(U\widetilde{CL}) = \frac{1}{2}\int_0^1[(UCL)_\alpha^L + (LCL)_\alpha^U]d\alpha = 1.50$$

By using MATLAB the inverse function of L(z) and R(z) of $(CL)_\alpha^L$, $(CL)_\alpha^U$, $(UCL)_\alpha^L$&$(UCL)_\alpha^U$ are obtain the membership function of

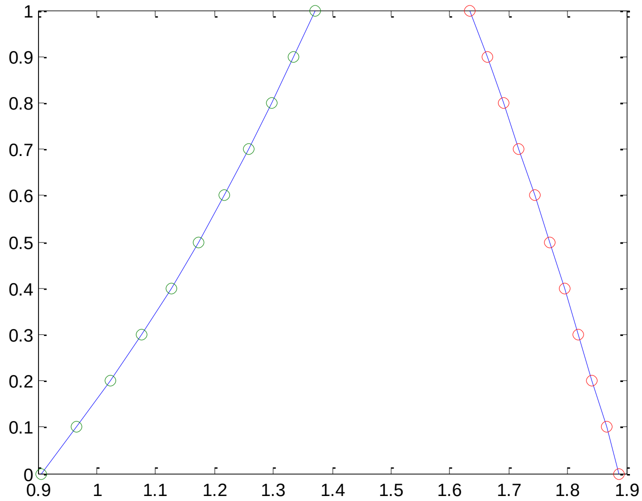
$\eta_{\widetilde{CL}}(\omega)$ and $\eta_{\widetilde{UCL}}(\omega)$ can stated as

$$\eta_{\widetilde{CL}}(\omega) = \begin{cases} L(z), & 0.9052 \le \omega \le 1.3705 \\ 1, & 1.3705 \le \omega \le 1.6347 \\ R(z), & 1.6347 \le \omega \le 1.888 \end{cases}$$

$$\eta_{\widetilde{UCL}}(\omega) = \begin{cases} L(z), & 23.9881 \le \omega \le 35.3177 \\ 1, & 35.3177 \le \omega \le 41.7164 \\ R(z), & 41.7164 \le \omega \le 46.1993 \end{cases}$$

Figure 2.1 (Trapezoidal CL (L))

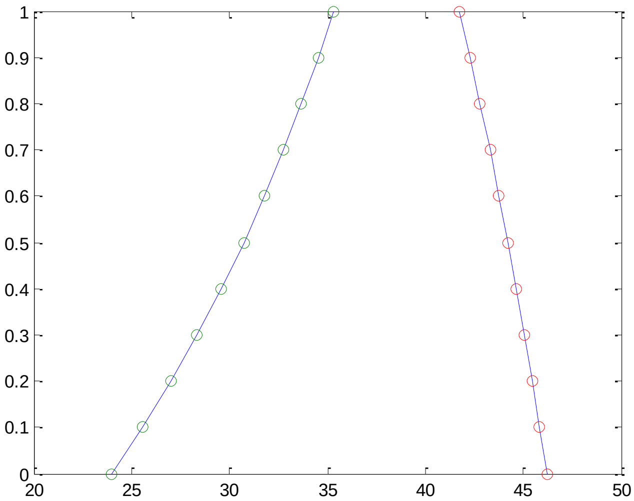
Values of α



Membership function of CL for expected system length L

Figure 2.2 Trapezoidal UCL(L)

Values of α

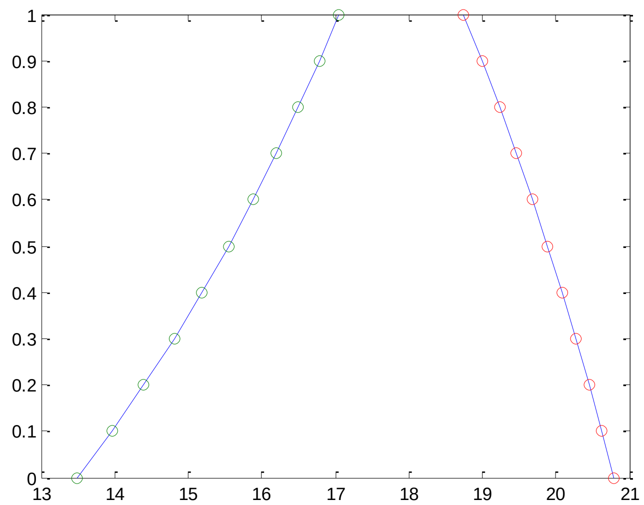


Membership function of UCL for expected system length L

The graph of the membership function of $\eta_{\widetilde{CL}}$ (ω) , $\eta_{\widetilde{UCL}}$ (ω) corresponding to α-cutsrelating to E(L) are shown in fig.2.1 and 2.2 respectively. And also given the graphs of the optimum threshold N* and variance (L).

Figure2.3 Trapezoidal(N)

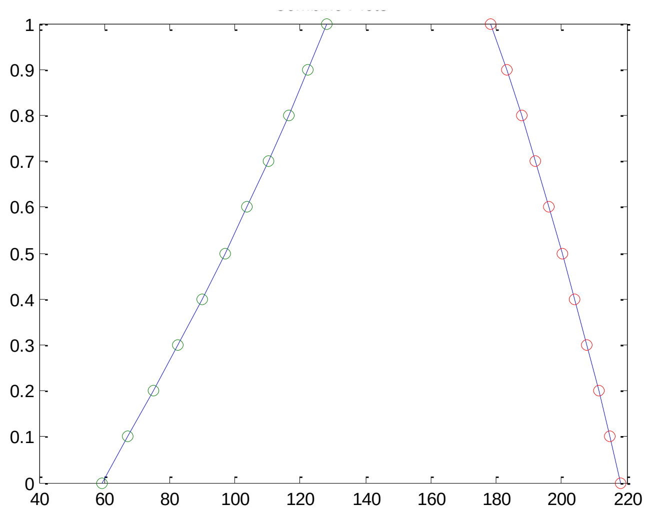
Values of α



Values of N

Figure 2.4 (Trapezoidal var(L))

Values of α



Variance (L)

Conclusion

In this paper, we constructed the expected system length (L) for an N-policy FM/FM/1 vacation queueing system with server start-up and time-out using fuzzy control chart and also constructed the membership function of triangular and trapezoidal fuzzy numbers.

References

[1] B.T. Doshi, Queueing systems with vacations, a survey Queueing Syst., (1986) 1, pp. 29-66.

[2] Oliver C. Ibe, M/G/1 Vacation Queueing Systems with Server Timeout, American Journal of Operations Research, (2015) 5, 77-88 Published Online March 2015 in SciRes. http://www.scirp.org/journal/ajor http://dx.doi.org/10.4236/ajor.2015.520

[3]K. Satish Kumar , K. Chandan and A. Ankama Rao, Optimal strategy analysis of N-policy M/M/1 vacation queueing system with server start up and time out,International Journal of Engineering Science Invention, (2017), ISSN (Online):2319-6734, Vol.6, issue11, 24-28.

[4] Y Saritha, K Satish Kumar and K Chandan,M^X /G/1 vacation queueing system with server timeout, International Journal of Statistics and Applied Mathematics, (2017), 2(5): 131-135

[5]J.J. Buckely, Elementary Queueing theory based on possibility theory, fuzzy sets and systems,(1990), 37, 43-52.

[6]S.P. Chen, A bulk Arrival Queueing Model with Fuzzy Parameters and varying Batch sizes, Applied Mathematical Modeling, (2006), 30, 920-929.

[7] Montgomery. D.C, Introduction to statistical quality control, 5th edition, John Wiley&sons, Inc., 2005

[8] A. Pandurangan & R. Varadharajan, "Construction of α-cut fuzzy $\overline{\overline{\tilde{X}}}$ - $\tilde{R}$ and $\overline{\overline{\tilde{X}}}$ - $\tilde{S}$ control charts using fuzzy trapezoidal number"www.arpapress.com/ IJRRAS, (2011), 9(1).

[9]. T.Poongodi and Dr.(Mrs)S.Muthulaksmi, Fuzzy control chart analysis of mean waiting time in M/M/Squeueing model,International Journal of Current Research,August, (2015), Vol 7, Issue, 08, pp.18988-18994,ISSN:0975-833X.

[10] T.Poongodi and Dr.(Mrs)S.Muthulaksmi,Fuzzy control chart for number of customers of Ek/M/1queueing model, International Journal of Advanced Scientific and Technical Research, (2015), issue 5, volume 3, ISSN 2249-9954.

[11] Mohammed Shapique.A,(2016) Analysis of Waiting Times in M/M/1 Queueing Model using Fuzzy Control Chart, the International Journal Of Science & Technoledge, (2016), Vol 4, Issue 1, ISSN 2321-919X,pp-16-22.

[12] L.A. Zadeh,(1965) Fuzzy sets. Information and Control,(1965), vol. 8, pp. 338-353.

[13] T.poongodi and S. Muthulakshmi, Control chart for waiting time in system of (M/M/S):(∞/FCFS) queueing model, IOSR Journal of Mathamatics (IOSR- JM), (2013), e-ISSN: 2278-5728. Valume 5, Issue 6, pp-48-53

[14] K Julia Rose Mary , P.Monica and S.Mythili,(2014)Performance Measures of FM/FM/1 Queueing System With N-policy, International Journal of Physics and Mathematical Science, (2014), ISSN: 2277-2111 Vol. 4 (3), pp. 5-10/Mary et al.