Journal of Namibian Studies, 34 S1(2023): 1447–1464 ISSN: 2197-5523 (online)

COGNITIVE SKILLS OF RURAL STUDENTS TO ACCESS QUALITY LEARNING OPPORTUNITIES IN THE MATHEMATICS CLASS

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Abstract:
The purpose of this article was to characterize the cognitive abilities of students belonging to a rural context to access quality learning opportunities in the mathematics class. We proceeded under Design-Based Research (IBD), focused on the development of theories about learning processes, twelve (12) students from grades six to ninth participated; between the ages of 12 and 17, from a Colombian rural area. Participant observation, video analysis and Hypothetical Learning Trajectories (THA) were used. The theoretical framework was Critical Mathematics Education and Teaching for Robust Understanding. As results, it was found that there is homogeneity in the levels of cognitive abilities of mathematical pre-knowledge, despite being students of different grades and ages. Besides, the participants are below their level of schooling. It was concluded that the low academic performance of rural students is mainly due to difficulties in accessing quality Learning Opportunities that exacerbate the low level of cognitive skills throughout their academic career. So, this group of students can be characterized as students in conditions of academic risk.
Keywords: Quality Learning Opportunity, cognitive skills, School Mathematics, rural students

Introduction:
Research in Mathematics Education is considered to be of high quality as long as it improves knowledge of the field and has a positive impact on the teaching and learning of mathematics in the classroom (Cai et al., 2020). In coherence with these ideas, there is one of the most important findings of education, which is that in order to learn, students must have opportunities to learn (National Research Council, 2001).

Through this lens, it can be analyzed that the Learning Opportunities that students in Colombian rural schools have had are bleak. In the same way, it can be seen from the different studies that have been carried out around Mathematics education in rural contexts, that sociodemographic and poverty conditions ARE causes of low performance and low results in standardized tests such as the ICFES tests, PISSA tests and TIMSS. These research results, for this group of students, have been widely reported as low achievement, rather than lack of opportunities provided by education systems (De Araujo & Smith, 2022).

It should be noted that although these conditions of vulnerability are factors that obviously affect the learning of mathematics, they are not necessarily the main causes of these results (Prediger and Buro, 2021). Therefore, the purpose is to characterize the cognitive abilities of rural students to access quality learning opportunities in the mathematics class.

This gap is embodied in the predominantly deficient vision of Rural Mathematics Education (RME) in the literature (Murphy, 2021). Eventually, international reports show that rural students have a lower performance than their urban counterparts, this being the common characteristic in all educational systems in the world (Maass et al., 2019a; Jorgensen, 2018; Adler, 2021). This inequality has practically been maintained over time (Wilson, 2021). For their part, Valero & Skovsmose (2012) discuss that all students in the world should have the possibility of learning mathematics, to enable the transformations of their living conditions.

For its part, the OECD (2021) maintains that all students should learn and have the opportunity to learn to think mathematically. For this, mathematical reasoning and problem-solving processes in context are essential, which is why they are the main elements of the PISA 2021 mathematics assessment (Maass et al., 2019b). In contrast, in Latin America, more than 50% of 15-year-old students do not reach the basic level of performance in Mathematics OECD (2016).

Similarly, the results of the PISA tests in Colombia show the gaps in the access to learning opportunities of rural students in relation to urban ones, private educational establishments registered a better performance in mathematics than official urban and rural educational establishments. (ICFES, 2019). The global trend shows that in recent years the mathematics results of children in rural areas have remained below that of their counterparts in urban areas.

In this sense, there is evidence that when children in rural schools have been offered the opportunities to learn, they perform even significantly better than their counterparts in urban schools (Murphy, 2019; 2021). Similarly, Avery (2013) points out that context-based education offers them a brilliant alternative. Online with these ideas, the student from the rural school environment presents higher levels of interest and curiosity in science (Morris et al., 2021).

Indeed, the Special Rural Education Plan of the Ministry of National Education of Colombia MEN (2018), indicates that in this population the results of the Saber 11 Tests are lower, especially in the area of mathematics, which limits access to higher education. At the regional level, the Sucre Departmental Development Plan 2020-2023 (PDD, 2020) shows that students from rural schools present low academic performance in mathematics, in addition to increased dropouts, low results in standardized tests, decreased possibilities of admission to higher education and high repetition rates (PDD, 2020).

From a preliminary study, it was observed that children present difficulties in mastering prior knowledge of mathematics, in solving basic problems, and in developing simple procedures, according to the requirements of the Ministry of National Education (MEN, 1998; 2006; 2016). It should be noted that children are creative, have good intuition, and offer a positive response to both cognitive and affective scaffolding, showing motivation to learn to solve problems. For all the above, the research problem was raised How are the cognitive abilities of rural students characterized to access quality learning opportunities in the mathematics class?

Theoretical framework

This chapter presents the theoretical framework to characterize the access to Quality Learning Opportunities of school mathematics for students from a Colombian rural context. This was elaborated from systematic reviews of the literature around thematic perspectives: Critical Mathematics Education (CME); The Teaching Framework for Robust Understanding (TRU); Quality Learning Opportunities from the Mathematics class (OAC). Finally, a list of concepts that have been used throughout the research is presented: for example, what is assumed from this research for conceptual understanding, procedural fluency, problem solving, main cognitive abilities (pre-knowledge of mathematics, attention, language competence and metacognition).

Critical Mathematics Education (CME) It is assumed as theoretical support, since it tends towards a curriculum that addresses social inequalities and is committed to the formation of subjects with critical citizenship and social justice. What is a felt need of rural students. The EMC encourages the use of pedagogies that can lead to a more equitable and just mathematics education for all students, (Skovsmose 1999, 2011, 2014, 2016, Ravn & Skovsmose, 2019; Skovsmose & Valero 2012; Valero & Skovsmose, 2012). From this framework, it is expected to analyze research that documents successful experiences, based on the movement through the different learning environments proposed in the EMC Framework.

Learning environments from Critical Mathematics Education

To make mathematics meaningful to students, there are no simple principles to apply (Skovsmose, 2000). Correspondingly, from the EMC six possible types of Learning Environments have been specified, which enhance the learning of mathematics, which result from the crossing of the type of references (pure mathematics, to a semi-reality or to a real life situation) and of the forms of organization of the activity in the class (the paradigm of the exercise and the research scenarios) (Skovsmose, 2000; 2011), see Table 1.

Table 1. Learning Environments proposed by Critical Mathematics Education

	Forms of organization of the activity of the students	Paradigm The exercise	Scenarios of investigation
Reference type	Math pure	(1)	(2)
	Semireality	(3)	(4)
	Situations of the real life	(5)	(6)

Source: Skovsmose (2000, p.10)

Teaching for Robust Understanding (TRU) framework

Schoenfeld (2016) provides quality criteria for a Learning Environment, these depend on the degree to which the learning environment provides opportunities for students in the five dimensions: (1) The richness of disciplinary concepts and practices (content) available to learn, (2) Understanding and productive effort, (3) Meaningful and equitable access to concepts and practices, (4) Means to build positive disciplinary identities through presentation, discussion, and refinement of ideas and (5) Sensitivity of the environment towards their way of thinking.

Table 2. The Five Dimensions of Powerful Classrooms (Schoenfeld, 2016)

The Content	**Cognitive Demand**	**Equitable Access to Content**	**Agency, Authority and Identity**	**Formative Assessment**
The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful disciplinary thinkers. Discussions are focused and coherent, providing opportunities to learn disciplinary ideas, techniques,	The extent to which students have opportunities to grapple with and make sense of important disciplinary ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to	The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core disciplinary content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no	The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations about disciplinary ideas, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and	The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Pow erful instruction "meets students where they are" and gives them opportunities to deepen their understandings.

and perspectives, make connections, and develop productive disciplinary habits of mind.	demanding. The level of challenge should be conducive to what has been called "productive struggle."	matter how rich the content: all students need to be involved in meaningful ways.	the development of positive identities as thinkers and learners.	

Source: Shoenfeld (2016, p.3)

The TRU (Teaching for Strong Understanding) framework answers the research question What are the attributes of equitable and robust learning environments, in which all students are supported to become knowledgeable, resourceful and flexible disciplinary thinkers? The author points out that the answer depends on what we know as teachers and researchers, and describes it synthetically in Table 2 (Schoenfeld, 2016, p.3-4). The fundamental claim underlying the TRU Framework is that a group's performance on the five TRU dimensions is positively related to preparing students as flexible, knowledgeable, and resourceful thinkers and problem solvers. The results, then, can be correlated with student performance on robust measures of reasoning and problem-solving ability.

Hypothetical Learning Trajectories

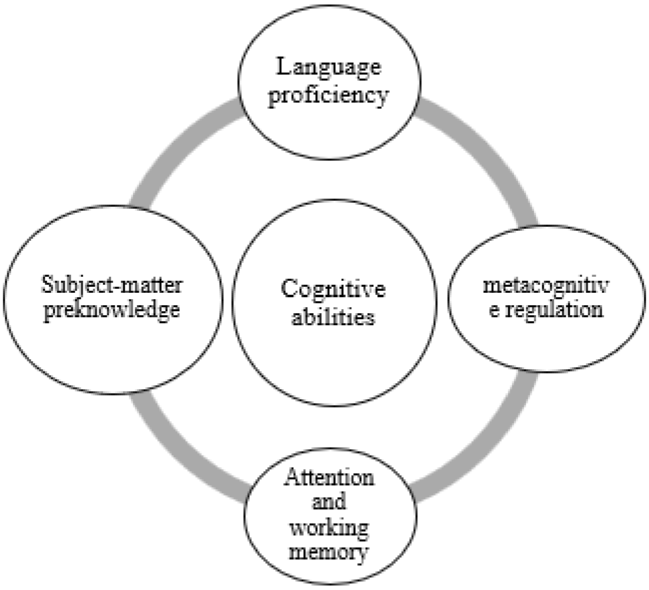
The construct about Hypothetical Learning Trajectories (THA) is a theoretical model for the design of mathematics teaching. It consists of three components: a learning goal, a set of learning tasks, and a hypothetical learning process (Simon, 1995). The recent taxonomy of Lobato & Walter (2017) on THAs allows us to see differences and similarities in the interpretation of THAs. Which supported making the decision to configure the THA based on a combination of the construct of Simón (1995) and Clements & Sarama (2004). This trajectory must develop both mathematical content and general mathematical processes such as problem solving, reasoning, communication, connections, and representation; specific mathematical processes such as organizing information, patterns, and habits of mind such as curiosity, imagination, inventiveness, persistence, willingness to experiment, and sensitivity to patterns (Clements, 2004, p. 57, cited by Clements & Sarama, 2014).

Conceptualization of the access of rural students via cognitive skills

To characterize the access to Quality Learning Opportunities of students belonging to Colombian rural schools, it was necessary to follow a conceptualization of inclusion. For this purpose, the broader definition of inclusive education of UNESCO is followed, which refers toall thestudents and their `different needs and abilities'' (UNESCO 2009, 18). It is notable and functional that the UNESCO definition refers not only to the demographic background of students (such as immigrant background, gender, and socioeconomic status) but also to theircapabilities.Because many demographic backgrounds have only an indirect influence on subject learning processes, abilities (and affective characteristics) influence subject learning processes more directly and can be better fostered (Prediger & Buro, 2021).

As an example, taking into account the needs of rural students may involve improving their learning of additive or multiplicative structures. Referencing skills rather than demographic background allows for easy expansion to more students. The selection suggested by Pressley et al. (1989) cited by Prediger and Buro (2021) about the most important cognitive skills that have been shown to have an impact on successful learning at school see Figure 1. For his part, Rojas (2023) in his doctoral thesis included mastery skills affective, motivation, selfconcept, related to learning. In this article, one of the four cognitive abilities will be limited: mathematical preknowledge and beliefs, attitudes and emotions.

Figure 1. Major student abilities that can have an impact on successful learning (adapted by Prediger et al., 2019)



Student at academic risk

It is conceived as “one who is in danger of not completing their education with an adequate level of skills. Risk factors include low achievement, grade retention, behavior problems, poor attendance, low socioeconomic status, and attendance at schools with large numbers of poor students” (Slavin & Madden, 1989, p.4, cited by Prediger 2022, p.14). Those ideas are still valid today, as research in mathematics education (and special education) has identified typical mathematics learning needs for at-risk students. While early research primarily emphasized the relevance of underlying basic skills to current issues, a growing consensus emerged that skills should be intertwined with understanding basic concepts (Kilpatrick et al., 2001; in Prediger et al., 2022)

Methodological design

This chapter describes the process and results of implementing Design-Based Research (IBD). Especially design experiments, due to their two dual objectives: 1) design and improve teaching and learning devices for classrooms and 2) generate theoretical contributions through empirical research to understand teaching and learning processes of a specific content. (Prediger, 2019; Gravemeijer & Prediger, 2019). In line with these dual objectives, it was intended to generate Quality Learning Opportunities so that each student in a rural context reaches their optimal personal level in mathematics (THA of fractions and percentages). From the configuration of learning environments (AA) and the movement between them (Skovsmose, 2000), that are generated and refined through design experiments in iterative cycles. In parallel, a methodology is built to generate quality learning opportunities for each student and a definition of OAC for each student (educational research: theory generation).

The type of research is qualitative. Given the epistemological position assumed, SE configured the research question inductively, through the identification of a real instructional problem, concerning inequality and inequity in access to Quality Learning Opportunities experienced by children of the Colombian and world rural contexts. The approach is phenomenological and socio-critical, in the sense that in addition to seeking descriptions, explanations and inferences, based and significant enough of the study phenomena, it was intended to establish relationships between events, derive useful and convincing clarifications, make novel findings, etc., to finally help with social transformation. Consequently, a collaborative social approach is chosen, which seeks to generate collaborative, independent, and autonomous processes of information search, understanding, and transformation of practices related to Mathematics Education in rural contexts, seeking their improvement, based on assuming shared responsibilities (Camargo, 2021).

Phases of Research Based on Design (IBD)

In the investigation, the phases of Design-Based Research (IBD) were implemented: which generally consists of cycles of three phases each: Preparation and design, Teaching experiment and Retrospective analysis of the data. (Cobb et al., 2000). The models proposed by (Bakker & Van Eerde, 2015) were deepened; Gresalfi, 2015; Prediger et al. 2012 in Gravemeijer & Prediger, 2019). It should be noted that for the development of the design, the use of Hypothetical Learning Trajectories (THA) was useful during all phases of IBD as a design and research instrument.

Participants

The participants were made up of 12 students who in 2021 were in grades six through ninth grade, aged 13 to 17 years. All students reside in La Floresta village. In addition, the principal investigator is the one who directs the instruction. It is a small group in quantity that is called a laboratory (Prediger, 2019). It should be clarified that for some activities guest teachers participated, to address activities of metric and geometric thinking, Numerical Thinking, and activities related to Statistics - Census (Excel). They know the research closely, they also acted as observers and evaluators using the TRU rubric.

Techniques

Observation (participant): Observations of interactions in the classroom, systematization and analysis of the interaction of the triad: learning activities - students - teacher. Analysis of student production and ways of approaching class work, analysis of videos and audio lessons. In addition, the collection of information about the perspective of each student played an important role (Clarke et al., 2006 in Skovsmose & Borba, 2004).

Open and semi-structured interviews: interviews were applied first to each individual and group, to parents, to inhabitants of the village who can participate with their experience and knowledge in the configuration of learning environments that are likely to generate opportunities quality learning for every student. One of the tenured mathematics professors at IE Palmira was interviewed. With the objective of analyzing her experience through the investigation of successful and unsuccessful experiences that they have had in their teaching practice.

video analysis: the content of the videos was analyzed qualitatively, following some of the phases of the Planas (2006) model. It is highlighted that the face-to-face and participant observation of the class sessions allowed us to have a detailed image and deep understanding of what happened.

Instruments: Basic knowledge tests, quizzes on operations with whole numbers, problems involving additive and multiplicative structures and fractions, worksheets, THAs, and notebooks. Rubrics and questionnaires addressed to the students were applied: assessment of the activities, affective domain, general observation guide "Observe the Lesson Through the Eyes of the Student" (Marco TRU, Shoenfeld, 2016). Also, Rubrics addressed to three teachers were applied: rubric characterization of tasks. Source: Ni et al. (2018), problem characterization rubric, adapted from Cai et al. (2023) and TRU rubric for summative evaluation.

Analysis Categories
In this article, two categories of analysis related to the Main skills of students that can have an impact on successful learning are considered (adapted by Prediger et al. (2019): mathematical pre-knowledge and metacognitive regulation, see table 3. For a holistic characterization of students in a rural Colombian context, it is necessary to consider both cognitive skills, as well as affective domain skills and emotional intelligence, Rojas (2023).

Table 3. Categories of analysis

Categorías	Subcategorías	Descripción
Cognitive abilities	Subject-matter preknowledge	Mathematics pre-knowledge is very relevant for successful later learning: it comprises not only basic skills, but also the understanding of basic concepts (Prediger y Buro, 2021).
	Metacognitive regulation	The student shows availability (ability and willingness to get involved) in different problem solving and formulation tasks (tasks whose difficulty ranges from moderate to high) in their different types, formats and contexts, recognizing that their decision to make a productive effort will help them develop skills and to achieve the proposed goal (Rojas, 2023)

Source: self-made

Results and analysis

Subject-matter preknowledge
As a result of the first interventions and interactions with the children, a characterization of the cognitive abilities referred to mathematical preknowledge was achieved. We proceeded by analyzing the responses of the students to the evaluative tests on basic operations with integers, understanding of fractions as part of the whole, notions of magnitude, quantity, measure and units of measure, as well as problem solving.
Through the analysis it was possible to observe distinctive features of the students of the Colombian rural schools; such as that no significant difference was found in the domain of basic mathematical knowledge, despite being at different grade levels (sixth to ninth grade). It can be seen,

for example, in Figure 2, the response of a ninth grade student compared to the response of a sixth grade student, presents the same level of mastery.

Figure 2. Respuestas de estudiantes frente a la prueba de conocimientos de operaciones aritméticas básicas

Niño 9º grado • 3^2 $3+3=6$ = $3X3=9$	Una niño de sexto grado $3^2=6$

In the successive analysis of the results, it was possible to make a more exhaustive characterization of the students' abilities, which is reflected in Table 4. This is an adaptation of the standards by grade given by the National Council of Mathematics Teachers. (NCTM, 2020) allowing to observe that rural children are several years below their level of schooling, in terms of previous mathematical knowledge. Thus, there are children who do not master basic processes, for example, to estimate and measure; as well as represent and solve problems that involve multiplication and division, being these processes of second and third grade of primary school.

Table 4. Level of students according to the Standard by grade.

Grado	Estándar por grado	Level Low
2º	Measure and estimate lengths in standard units	x
3º	Represent and solve problems involving multiplication and division	x
3º	Geometric Measurement: understand area concepts and relate area to multiplication and addition	x
4º	Understand decimal notation for fractions, and compare decimal fractions	x
5º	Perform operations with multi-digit integers and decimals to hundredths	x
5º	Using equivalent fractions as a strategy to add and subtract fractions	x
6º	Understand the concept of proportion and use proportional reasoning to solve problems	x
6º	Reason and solve equations and inequalities of one variable	x
7º	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers	x
7º	Analyze proportional relationships and use them to solve real-world and mathematical problems	x
8º	Understand the connections between proportional relationships and linear equations	x

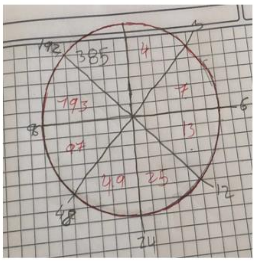
Source: self-made

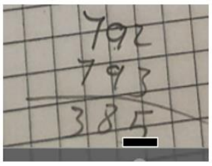
Metacognitive regulation

Although the students presented weaknesses in their mathematical pre-knowledge skills, it should be noted that, when addressing non-routine problems (Ni et al., 2018) that promoted reasoning processes and were not limited to the application of algorithms, the students They showed availability (ability and willingness to get involved) recognizing that their decision to make a productive effort will help them develop skills and achieve the proposed goal (Rojas, 2023).

Thus, they were able to solve problems such as the case of the puzzle, illustrated in figure 3. It was conjectured that they would provide a solution using the power of 2, but it was evidenced that they presented difficulties with the potentiation; however, the children used alternate solution ways, showing thus, elements of metacognition. As a result, in part, of the intervention, in which the Socratic method was used as a nuance of instruction and the movement through different Learning Environments (Skovmose, 2000), the author, who had the role of teacher through questions, Careful not to give her the answers, she directed the children's attention to finding patterns to finish the series. After spending more than two hours thinking, a seventh grader managed not only to give a correct answer, but also to propose a new challenge following the sequence.

Figure 3. Production of students on solving problems with the absence of mathematical content and use of mathematical reasoning.





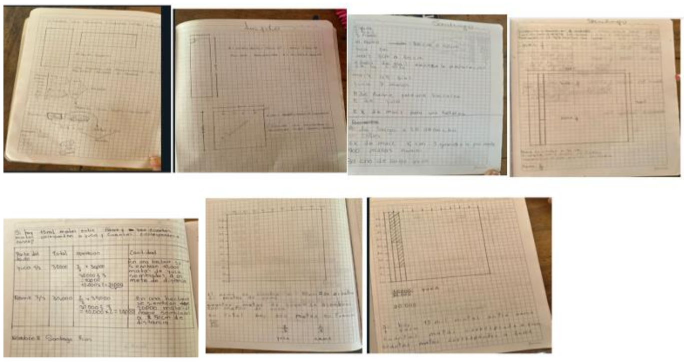
Solution de Ever (7º)

Quality learning opportunities permeated by the movement through the different Learning Environments of Critical Mathematics Education

These Quality Learning Opportunities tend to enhance cognitive skills in rural students. Next, the development of tasks of a Hypothetical Learning Trajectory (THA) is illustrated, which mainly emphasize the relevance that skills must be intertwined with the understanding of basic concepts (Kilpatrick et al., 2001; in Prediger et al., 2022). This THA consists of tasks related to problem solving involving scale drawings of geometric figures, including calculating actual lengths and areas from one scale, drawing and reproducing a scale drawing at another scale (NTCM, 2020). To promote the learning of fractions and proportional reasoning in solving problems related to crops to estimate, for example, the amount of seeds required to grow cassava and yams.

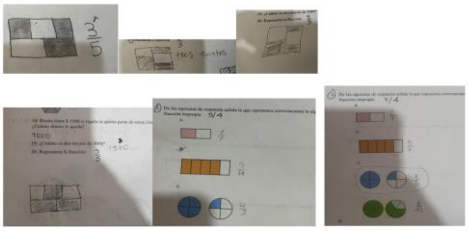
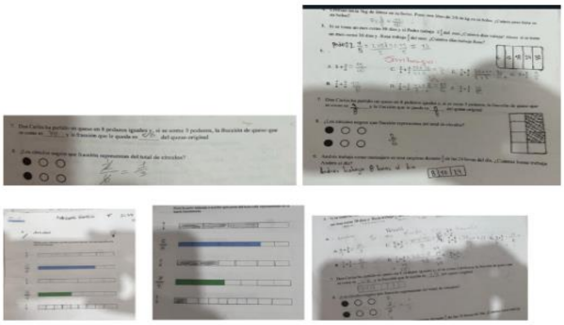
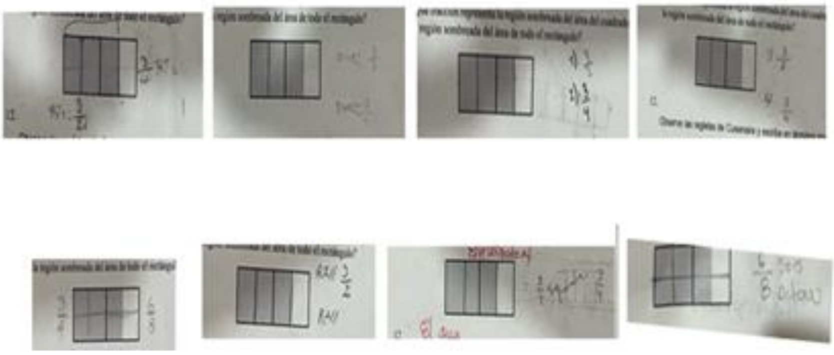
Description of the development of the tasks: The children interviewed several peasants to obtain information about the measurements of the sides of the cultivated area, the number of streets that make up a certain crop, how far is one street from another, the distance between one seed and another. Thus, for example, the first problem consisted of using the information obtained in the interviews with the farmers to estimate how many seeds he needed to cultivate a hectare of 100 mx 100 m. and from there, make estimates for smaller land areas.
One way in which the fractions were inserted was, for example, to answer: if there were 15 thousand plants between yams and yucca, how many plants correspond to yucca and how many to yams? See photos with some of the answers. They also made drawings to scale see Figure 4. It was observed how the children show greater robustness in their reasoning, to the point that they extrapolate information to solve inherent problems. In correspondence with the ideas of Balda (2018), who in her doctoral thesis characterizes the frequent actions that arise from her work with the School Garden, the author calls them: recurring mathematical manifestations, are: comparison, measurement, classification, selection, estimation, counting, anticipation and approximation.

Figure 4. Student responses to problems about cultivation and interaction with farmers.



Hypothetical Trajectory of Learning (THA) on problems with fractions
Difficulty that the students presented to understand the proposed problems, the movement between Learning Environments was made and the formative evaluation was implemented (Shoenfeld, 2016). The progress of the students in the levels of conceptual understanding about fractions is evidenced with a sequence of tasks of a material sensitive to language (Prediger, 2019). Evidence is presented of how the students were progressing in the learning trajectory through the tasks, in the different levels, see figure 5.

Figure 5. Student responses on fraction knowledge tests.

Niveles	**Evidencia**
Level 1: Students cannot identify or represent fractions. They do not recognize that the parts into which the whole is divided must be equal in area (they may be different in shape, but they must be equal in area)	
Level 2. Students can identify and represent proper fractions	
Level 3 (Students can identify and represent improper fractions) What fraction does the shaded region represent with respect to the left square? And what fraction does the shaded region represent with respect to the area of the entire rectangle? 	

The problem of level 3, figure 5, was a challenge for the children, which allowed them to maintain a productive effort during the class. Regarding the solution, only 5 of the 12 students managed to understand at a high level: it took them more than an hour. This problem was interesting in the sense that, in addition to maintaining the cognitive demand (Shoenfeld, 2020), I allowed them to put into practice metacognitive processes about how persevering and persisting made progress in mathematical thinking possible (Shoenfeld, 2016), so In the same way, by making students' thoughts public, they manage to transform beliefs about the importance of immediacy when solving problems, they understand the importance of perseverance to achieve learning goals (Rojas, 2023).

Conclusions

When students are offered quality learning opportunities which are permeated by the movement through the six learning environments of Critical Mathematics Education, each student achieves advance levels of mathematical thinking, through interweaving a conceptual understanding with the procedural being this the trajectory to follow to achieve leveling in the previous mathematical knowledge. Thus, the movement for the different AA of Critical Mathematics Education aimed at generating equitable and solid learning environments, in which all students have the opportunity to learn mathematics.

He used the Teaching for Robust Understanding Framework (TRU) with its five dimensions as an analytical lens, and the movement through the different Learning Environments generates a new Environment of Learning Environments, which promotes the attributes of equitable and robust learning environments, in which each of the students was able to access a Quality Learning Opportunity. So, this new Learning Environment provides each student with access to mathematical content, maintains the student's Cognitive Demand, favors Equitable Access to Mathematics, and promotes Availability, Mastery and Identity, and through the continuous practice of Formative Assessment.

On the other hand, four major problems of rural education in Colombia stand out in contrast to education in urban areas: high dropout rates and low secondary education graduation rates, low academic performance of students, high rate of overage students in rural areas and the low rate of access to higher education and high dropout rates. For all of the above: the child from a rural context with the levels described can be characterized as a student in conditions of academic risk.

Finally, depending on the learning opportunities offered to children in these rural settings, the gaps can be perpetuated (Yang & Strietholt, 2018). Consequently, it is pertinent to reflect on how all children can learn and progress at their individual level and at the same time learn with each other (Höveler, 2019). It should be emphasized that the problem of equitable access to Quality Learning Opportunities for rural students cannot be studied only from a narrow conceptualization, that is, taking into account only the demographic aspect and their socioeconomic situation. Rather, taking into account the needs, both cognitive and affective abilities of this group of students, since these aspects directly influence the learning processes (Prediger & Buró, 2021; Rojas, 2023).

REFERENCES:

Adler, J. (2021). Levering change: the contributory role of a mathematics teaching framework. ZDM - Mathematics Education, 53(6), 1207–1220. https://doi.org/10.1007/s11858-021-01273-y

Avery, L. M. (2013). Rural Science Education: Valuing Local Knowledge. Theory into Practice, 52(1), 28–35. https://doi.org/10.1080/07351690.2013.743769 Bakker, A., & Van Eerde, D. (2015). An introduction to design-based research with an example from statistics education. Approaches to qualitative research in mathematics education: Examples of methodology and methods, 429-466.

Bakker, A., & Van Eerde, D. (2015). An introduction to design-based research with an example from statistics education. Approaches to qualitative research in mathematics education: Examples of methodology and methods, 429-466.

Balda, P. (2018). Una epistemología de usos en torno a lo proporcional: un estudio socioepistemológico en el contexto de la huerta escolar (Issue February). Universidad Santo Tomas

Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V., Cirillo, M., Kramer, S. L., Hiebert, J., & Bakker, A. (2020). Maximizing the quality of learning opportunities for every student. Journal for Research in Mathematics Education, 51(1), 12–25. https://doi.org/10.5951/jresematheduc.2019.0005.

Camargo Uribe, L. (2022). Estrategias de investigación cualitativa en educación matemática.

Clements, DH y Sarama, J. (2014). Learning and Teaching Early Math. Routledge.

Cobb, P., Confrey, J., Disessa, A., Lehrer, R., & Schauble, L. (2003). Design Experiments in Educational Research. Educational Researcher, 32(1), 9–13. https://doi.org/10.3102/0013189X032001009

De Araujo, Z., & Smith, E. (2022). Examining English language learners’ learning needs through the lens of algebra curriculum materials. Educational Studies in Mathematics, 109(1), 65-87.

Dewey, J. (1938). Experiencia y educación Libro de John. Estados Unidos: Kappa Delta Pi.

Ernest, P. (2010). Reflections on Theories of Learning. In B. Sriraman, L. English, Editors, & Theories (Eds.), Theories of Mathematics Education (pp. 35–47). https://doi.org/10.1007/978-3-642-00742-2

Gravemeijer, K., & Prediger, S. (2019). Topic-specific design research: An introduction. Compendium for early career researchers in mathematics education, 33-57.

Gresalfi, M. S. (2015). Designing to support critical engagement with statistics. ZDM - Mathematics Education, 47(6), 933–946. https://doi.org/10.1007/s11858-015-0690-7

Höveler, K. (2019). Inclusive Mathematics Education. Inclusive Mathematics Education. https://doi.org/10.1007/978-3-030-11518-0 https://www.nctm.org/uploadedFiles/Research_and_Advocacy/NCTM_NCSM_Moving_Forward.pdf

Jorgensen, R. (2018). Building the mathematical capital of marginalised learners of mathematics. ZDM - Mathematics Education, 50(6), 987–998. https://doi.org/10.1007/s11858-018-0966-9

Lobato, J., & Walters, C. D. (2017). A taxonomy of approaches to learning trajectories and progressions. Compendium for research in mathematics education, 74-101.

Maass, K., Geiger, V., Ariza, M. R., & Goos, M. (2019a). The Role of Mathematics in interdisciplinary STEM education. ZDM - Mathematics Education, 51(6), 869–884. https://doi.org/10.1007/s11858-019-01100-5

Maass, K., Geiger, V., Ariza, M. R., & Goos, M. (2019b). The Role of Mathematics in interdisciplinary STEM education. ZDM - Mathematics Education, 51(6), 869–884. https://doi.org/10.1007/s11858-019-01100-5

Ministerio de Educación Nacional. (1998). Serie Lineamientos Curriculares. http://www.mineducacion.gov.co/1759/articles-339975_recurso_6.pdf

Ministerio de Educación Nacional. (2006). ESTÁNDARES BÁSICOS DE COMPETENCIAS EN MÁTEMÁTICAS. In Magisterio (pp. 46–48). https://www.mineducacion.gov.co/1621/articles-116042_archivo_pdf2.pdf

Ministerio de Educación Nacional. (2016). Derechos Básicos de Aprendizaje DBA versión 2. http://iedar.edu.co/DBA/DBA MATEMATICAS 2 EDISION.pdf

Morris, J., Slater, E., Fitzgerald, M. T., Lummis, G. W., & van Etten, E. (2021). Using Local Rural Knowledge to Enhance STEM Learning for Gifted and Talented Students in Australia. Research in Science Education, 51, 61–79. https://doi.org/10.1007/s11165-019-9823-2

Murphy, S. (2019). School location and socioeconomic status and patterns of participation and achievement in senior secondary mathematics. Mathematics Education Research Journal, 31(3), 219–235. https://doi.org/10.1007/s13394-018-0251-9.

Murphy, S. (2021). Mathematics success against the odds: the case of a low socioeconomic status, rural Australian school with sustained high mathematics performance. Mathematics Education Research Journal. https://doi.org/10.1007/s13394-020-00361-8

National Council of Supervisors of Mathematics & National Council of Teachers of Mathematics. (2020). Moving forward: Mathematics learning in the era of COVID-19. NCTM.

National Research Council, & Mathematics Learning Study Committee. (2001). Adding it up: Helping children learn mathematics. National Academies Press.

Ni, Y., Zhou, D. H. R., Cai, J., Li, X., Li, Q., & Sun, I. X. (2018). Improving cognitive and affective learning outcomes of students through mathematics instructional tasks of high cognitive demand. The Journal of Educational Research, 111(6), 704-719.

OCDE. (2021). Pisa 2021 Mathematics Framework (Second Draft). March 2017, 1–47. https://pisa2021-maths.oecd.org/files/PISA 2021 Mathematics Framework Draft.pdf

Planas, N. (2006). Modelo de análisis de videos para el estudio de procesos de construcción de conocimiento matemático. Educación matemática, 18(1), 37-72.

Prediger, S. (2019). Theorizing in design research: Methodological reflections on developing and connecting theory elements for language-responsive mathematics classrooms. Avances de Investigacion En Educacion Matematica, 15, 5–27. https://doi.org/10.35763/aiem.v0i15.265

Prediger, S., & Buró, R. (2021). Fifty ways to work with students' diverse abilities? A video study on inclusive teaching practices in secondary mathematics classrooms. International Journal of Inclusive Education, 0(0), 1–20. https://doi.org/10.1080/13603116.2021.1925361

Prediger, S., Dröse, J., Stahnke, R., & Ademmer, C. (2022). Teacher expertise for fostering at-risk students' understanding of basic concepts: conceptual model and evidence for growth. Journal of Mathematics Teacher Education, 1-28.

Quabeck, K., Erath, K., & Prediger, S. (2023). Measuring interaction quality in mathematics instruction: How differences in operationalizations matter methodologically. The Journal of Mathematical Behavior, 70, 101054.

Rojas, S. (2023). Hacia la generación de oportunidades de aprendizaje de calidad para cada estudiante en la clase de matemáticas en contextos rurales. [Tesis de doctorado no publicada]. Universidad Antonio Nariño. Colombia.

Schoenfeld, A. H. (2016). An Introduction to the Teaching for Robust Understanding (TRU) Framework.

Schoenfeld, A. H. (2020). Mathematical practices, in theory and practice. ZDM - Mathematics Education, 52(6), 1163–1175. https://doi.org/10.1007/s11858-020-01162-w

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. Journal for research in mathematics education, 26(2), 114-145.

Skovsmose, O. (1999). Hacia una filosofía de la educación matemática crítica.

Skovsmose, O. (2000). Scenarios de investigación1. Ema, 6, 3–26. http://funes.uniandes.edu.co/1122/1/70_Skovsmose2000Escenarios_RevEMA.pdf

Skovsmose, O. (2005). MEANING IN MATHEMATICS EDUCATION. In J. Kilpatrick, C. Hoyles, O. Skovsmose, & P. Valero (Eds.), MEANING IN MATHEMATICS EDUCATION (A.J. Bisho, Vol. 37, pp. 83–99).

Skovsmose, O. (2011). An Invitation to Critical Mathematics Education.

Skovsmose, O. (2016). Critical Mathematics Education: Concerns, Notions, and Future. In P. Ernest, O. Skovsmose, J. P. van Bendegem, M. Bicudo, R. Miarka, L. Kvasz, & R. Moeller (Eds.), The Philosophy of Mathematics Education ICME-13 Topical Surveys Series (Gabriele K, Vol. 44, Issue 8, pp. 9–13). https://doi.org/10.1088/1751-8113/44/8/085201

Skovsmose, O., & Borba, M. (2004). Research methodology and critical mathematics education. In P. Valero & R. Zevenbergen (Eds.), Researching the Socio-Political Dimensions of Mathematics Education (pp. 207–226). https://doi.org/10.1007/b120597

Valero, P., & Skovsmose, O. (2012). Educación matemática crítica Una visión sociopolítica del aprendizaje y la enseñanza de las matemáticas (Issue April).

Wilson, K. (2021). Exploring the Challenges and Enablers of Implementing a STEM Project-Based Learning Programme in a Diverse Junior Secondary Context. International Journal of Science and Mathematics Education, 19(5), 881–897. https://doi.org/10.1007/s10763-020-10103-8

Yang, K., & Strietholt, R. (2018). Does schooling actually perpetuate educational inequality in mathematics performance? A validity question on the measures of opportunity to learn in PISA. ZDM - Mathematics Education, 50(4), 643–658. https://doi.org/10.1007/s11858-018-0935-3