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Learning About The Rationale For Change With Contextualized Conceptual Cartoons

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Abstract

The teaching of calculus in the initial training of mathematics teachers allows the development of mathematical and didactic knowledge. The purpose of the study was to characterize the reflections of a group of turquoise preservice teachers when they interacted with a conceptual cartoon contextualized, as a didactic tool, in teaching the concept of derivate as a rate of change. The study used an interpretative approach to the implementation process of this didactic tool in three phases: a. personal reflections and questions that emerge from the cartoon, b. group discussion, and c. group analysis of the final cartoon vignette. The results show the didactic potential of the concept cartoon by favoring the formulation of questions by the preservice teachers, particularly the argumentation from the concepts of differential calculus. Likewise, the contextualized concept cartoon was a mediating narrative in the preservice teachers' dialogues, doing the constant search for understanding of the cartoon characters' reasoning logics possible.

Keywords: mathematical concepts; professional education; preservice teacher education; didactic; contextualized concept cartoons.

Introduction

When we look at the universe, almost everything is constantly moving and changing. Calculus, on the other hand, is a branch of mathematics that involves the study of rates of change. With the invention of Calculus, it was possible to describe how particles, stars, and matter move and change in real time. Scientists, astronomers, physicists, mathematicians, and chemists have used the concepts of calculus to determine the orbits of planets and stars and the path of electrons and protons at the atomic level. In addition to being used in different fields such as biology, physics, engineering, economics, statistics, medicine, and space travel, it is also used in ship design, geometric curves, and bridge engineering studies. In other words, Calculus allows mathematicians and engineers to make sense of motion and dynamic change in the changing world (Russell, 2020).

According to Hartter (1995), Calculus is among the essential courses in undergraduate mathematics programs and has two branches: differential and integral calculus. The differential part examines the rate of change (rates) of functions according to their variables, primarily through derivatives and differentials. The basis of the rate of change is found in everyday experiences such as growth and movement and is a fundamental organizing idea for relationships between varying quantities (Confrey & Smith, 1992; Smith Hauger, 1995, Stewart, 1993). It explains how two variables change about each other, quantitatively and qualitatively (Thompson, 1994b). With the concept of rate of change, it is possible to define which function family the function belongs to, which means it is possible to understand the function (Cooney et al., 2010). In addition, students' conceptual competence on the rate of change will effectively understand concepts such as functions, derivatives, integrals, and differential equations (Bezuidenhout, 1998; Rowland & Javanoski, 2004; Thompson, 1994b; White & Mitchelmore, 1996). Rate of change is the most decisive and comprehensive conceptualization of the concept of derivative, and it is often used in the interpretation of derivatives in real-world situations (Stroup, 2002; Thompson, 1994b). In addition, the rate of change is essential for conceptually understanding the multiple representations and interpretations of the derivative (Cooney et al., 2010; Kendal & Stacey, 2003; Zandieh, 2000).

When the literature is examined, it is stated that mathematics concepts are not easy for learners and teachers (Carlson et al., 2002; Coe, 2007; Orton, 1984; Ubuz, 2007). Almost every mathematics teacher has stories about their students' difficulties in mathematics, which are attributed to several shortcomings (Smith Hauger, 1995). For example, students lack knowledge about functions; they have difficulty constructing and manipulating algebraic expressions for relationships between quantities; they do know the essential graphic properties of function families and cannot move freely between function representations such as equations, value tables, and graphs. The concept of rate of change can also be said to be one of the basic concepts of mathematics with difficulty (Bezuidenhout, 1998; Carlson et al., 2002; Cooney et al., 2010; Coe, 2007; Gökçek & Açıkyıldız, 2016; Herbert & Pierce, 2008, 2012; Johnson, 2012 Teuscher & Reys, 2012; White & Mitchelmore, 1996; Thompson, 1994a). In particular, it is stated that although students can use the symbolic representation of the derivative correctly in practice, they cannot connect it with other mathematical concepts studied in previous years (Delos Santos & Thomas, 2005). Similarly, students have deficiencies in the interpretation of the derivative's rate of change; high school students and prospective teachers could not make sense of and answer the question of the rate of change of a quadratic function at a certain point; It is stated that students' knowledge of change rate concepts is limited to the operational level and problem contexts that are not related to daily life (Bingölbali, 2008; Orton, 1983; White & Mitchelmore, 1996, cited in Kertil, 2017).

Some of the difficulties that the researchers again put forward on the rate of change can be listed as follows. i) lack of knowledge about some basic concepts such as covariational thinking, slope, and function (Herbert & Pierce, 2008), ii) Students confuse the concept of rate of change with only the amount of change in the dependent variable (Thompson, 1994b; Zandieh & Knapp, 2006), iii) Teacher and students not being able to relate it to other contexts because they saw the concept of rate of change only in its limited context such as speed-time (Herbert & Pierce, 2008, 2012; Zandieh & Knapp, 2006). Recent studies on the rate of change concept show that these difficulties still exist (Bezuidenhout, 1998; Gökçek & Açıkyıldız, 2016; Herbert & Pierce, 2012; Johnson, 2012; Teuscher & Reys, 2012; Ubuz, 2007). Bingölbali (2008) interprets the situation as an epistemological problem since similar problems are experienced in different countries.

Despite the importance of calculus, one of the basic concepts, it should not be surprising that students have difficulty interpreting this concept because the essence of this concept requires having appropriate function knowledge (Smith Hauger,1995). states that there is a need to deepen our understanding of the concepts and how this knowledge can be used" (p. 635).

Smith Hauger (1995) points out that focusing solely on students' difficulties may mean missing part of the learning story. For this reason, although it is essential to know where students struggle with a particular concept, it emphasizes that knowing how students successfully solve problems with that concept may be more beneficial. Based on this idea, the study was designed with the idea that it would be helpful to see how pre-service mathematics teachers use their existing knowledge to support their thoughts about the rate of change and to improve their knowledge of the rate of change. It is expected that the study will contribute to the conceptual structuring the rate ofof change in teaching and learning environments in general. The study used contextualized concept cartoons to reveal pre-service mathematics teachers' thoughts about the rate of change.

Contextualized Concept Cartoons (CCC) are associated with the idea of creating alternative narratives that emerged in existing phenomenological research and allowed students to re-interpret their understanding of phenomena explained by science (Cely, G., Reyes, J., & Bustos, E., 2018; Naylor & McMurdo, 1990; Porras & Reyes, 2019; Reyes & Romero, 2017; Reyes, Romero & Bustos, (2018, 2019, 2020). CCC is a didactic tool that provides opportunities for new understanding through the research process and raises questions and experimental plans to test specific hypotheses while locating the necessary connections between real life and the scientific world. Therefore, the study aims to characterize the thoughts of a group of pre-service mathematics teachers when they interact with a contextualized concept cartoon as a didactic tool in teaching the concept of derivative as a rate of change. For this purpose, the following questions were asked directly to the pre-service teachers:

1. What questions arise from their interaction with the cartoon?

2. What comments did they make on the rate of change related to the context of the problem posed?

3. Can they establish a relationship between the rate of change and the derivative in the case of graphical representation?

The analysis of the information also allows evidence of some alternative ideas about the rate of change and its relation to the world of life.

Design of the study

Participants

The research participants were 20 future secondary school teachers - full-time students in the fourth year's teacher training program at the University of Marmara. They had already finished Calculus I, Calculus II, and Calculus III mathematics content courses. They were attending a course on Differential Equations at the time of the study. During the course, the participants had 45 minutes to work with contextualized concept cartoons (Fig. 1), discussing them in the classroom and responding in written form to related questions.

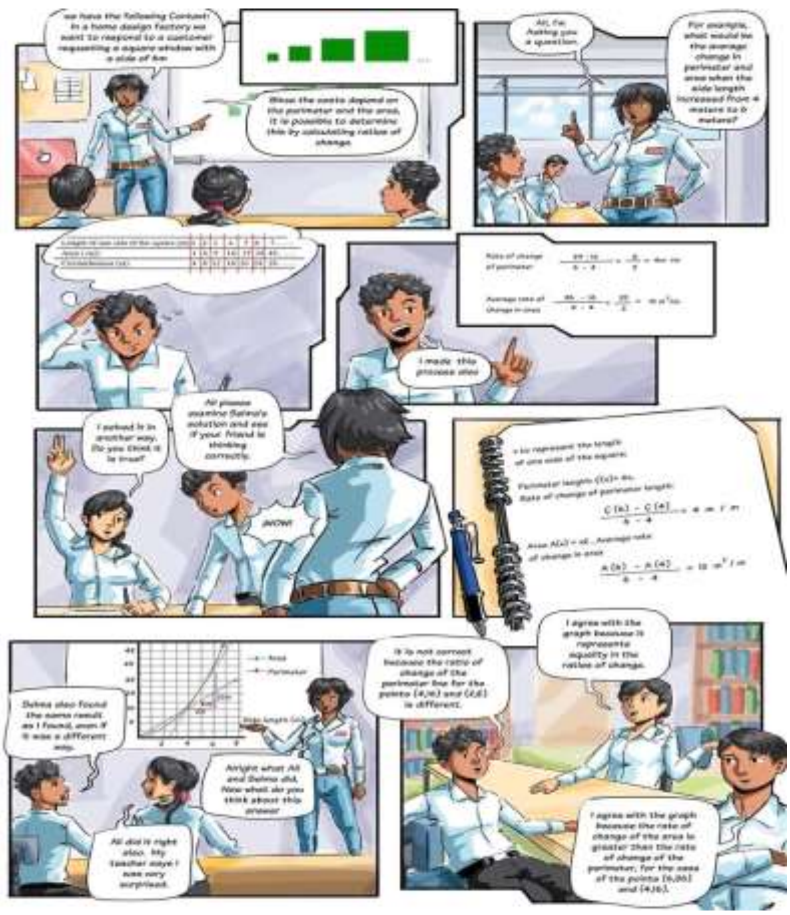

Figure 1. Contextualized Concept Cartoon used in the research.

The course of the study

In the data collection stage, the participants were asked to read the Contextualized Concept Cartoon; they worked on this task individually and wrote their thoughts about the cartoon. In the next step of the process, the students worked in groups discussing their personal experiences with the cartoon, and, finally, they analyzed specifically the final cartoon vignette that includes an R-2 graph.
The participants were asked the following question:

1. What questions arise because of their interaction with the cartoon?
2. What comments did they make on the rate of change related to the context of the problem?
3. Can they establish a relationship between the rate of change and the derivative in the case of graphical representation?

The qualitative collected data were processed using open coding (Creswell & Poth, 2018); during this process, we organized the information focused on the relationship between mathematical knowledge about the rate of change and derivates with personal and group reflections of the participants. As a result, the following code categories appeared relevant to our study:

1. Interpretation of reasoning: In this category, the questions are associated with the student's interest in knowing much more about the reasoning of the characters in the cartoon, the reasons for their decisions and solutions to the problem, and, in that sense, also the questioning of their differences.
2. Conceptual questioning: The students' reflections demonstrate the use of their knowledge of the topic, both to analyze the veracity of the processes carried out by the characters and to make conjectures about alternative situations using the data in the vignettes.
3. The contextualized concept cartoon as a didactic tool: here, questions refer to their interest in recognizing the importance of contextualized conceptual cartoons in their class and the value they place on this material in their learning. Additionally, comments are expressed on the cartoon's contribution to the generation of questions and its pedagogical value as an alternative to developing the topic.

Findings

In this section, the findings are presented according to the research sub-problems.

What questions arise from their interaction with the contextualized concept cartoon?

Within the scope of this question, it was found that pre-service teachers produced questions in different contexts because of their interaction with the contextual concept cartoons. This reveals that pre-service teachers can evaluate and interpret the situation from different perspectives. When the questions that arise are characterized, some teacher candidates emphasize questioning the reasons for the differences in the way the characters solve the problem, which has related to the category Interpretation of reasoning, for example,
“What is the difference between Selma's and Ali's solutions?”
“In the example he gave Ali, would the teacher have thought like Selma if she had asked a general question (e.g., how would the area change as the side length x increased?) instead of sample numbers?”
“Does Mehmet's answer apply to (1,1) and (2,4)?”
“What is the reason for Ali's wrong thinking?”
Regarding the conceptual reasoning category, some teacher candidates question the theoretical differences in the arguments between the characters, as follows:

Why did Selma and Ali solve the question correctly but could not find the right graph?
Why did Ali and Selma find the same result in this cartoon but different graphic interpretations?
How did Mehmet find the rate of change while commenting on the chart?
What concept did he use to conclude?
Does Ali have problems with graphical interpretation?
What did Selma mean by constancy in the rate of change?
What exactly does the graph mean?
Does the graph show equality in rates of change? (as Selma said)
Even though the graphic in this cartoon is clear enough, what is the reason for the disagreements among the students?

Regarding the reason for the change, the questions asked questions about the effects of the use of contextualized conceptual cartoons on the students' association of ideas or concepts and the structuring of their mathematical knowledge. For this reason, the trainee teachers became aware of the concept of rate of change that they wanted to question. Moreover, this concept is essential in shaping their conceptual structures by analyzing the reasons the characters in the contextualized conceptual cartoons argue from different perspectives. Some questions here are:
Are the concepts of the average rate of change and rate of change different?
While the graph shows the values the environment can take, are the values the area can take fully expressed?
Do the values in the graph comply with the area while covering the perimeter?
From this vignette, what are the benefits of this application for teachers and students?
From what point does the area affect more than the perimeter?

In these questions, trainee teachers are determined to question the different ways of representing the rate of change within the context and reflect on what inferences teachers and students should make from the information presented here.
Regarding the category of contextualized concept cartoons as a didactic tool, some teachers in training ask themselves:

Why are contextualized concept cartoons important?

Why are conceptual cartoons needed?

Why was the topic of rate of change chosen in the contextualized cartoon?

These questions reveal the students' interest in giving pedagogical meaning to using cartoons in a mathematics class in a practical or theoretical sense.

What comments did they make on the rate of change related to the context of the problem?

First category

Regarding the category of the interpretation of reasoning, the trainee teachers evaluated the explanations of the characters in problem-solving from a critical point of view and presented their opinions as follows:

Different representations of the rate of change (algebraic, table, graph) are used when calculating the rate of change.

Based on the same idea, Selma and Ali functionalized the concepts of perimeter and area as variables such as Ç(x), A(x).

Selma and Ali answered incorrectly because they needed to learn how to read the graph correctly.

It is emphasized that Ali got it wrong and that the rate of change in the environment is constant, and the rate of change in the area is variable.

Selma gave the incorrect answer; Considering that the rate of change of the area function is derivative, the rate of change of the area cannot be constant since it depends on the variable f(x) =2x.

Contrary to Selma and Ali, Mehmet uses the slope in the rate of change by interpreting the graph.

Mehmet makes a correct inference based on the fact that the function, when looking at the graph without using a point like Ali or a formula like Selma increases more.

I agree with Mehmet's answer. Because as the length of the edge increases, the rate of change of the area increases, and this is correctly shown in the graph.

I agree with Mehmet because the rate of change of the area is greater than the rate of change of the perimeter in all cases except when the side length is equal to 1.

Answer that the slope of the tangent drawn to the curved line of the field gives us the rate of change. It gives us the circumference in a straight line.

In light of these opinions, it can be said that the teacher candidates know the multiple representations of the rate of change and concluded that: the rate of change of the perimeter must be constant and the rate of change of the area must be variable. In addition, they realized that the rate of change is equal to the slope, and they established the relationship of the derivative to the rate of change and realized the different readings of the graph.

Nevertheless, one of the teacher candidates said, "When we examine the graph, we find that the rate of change of the area is 2x and the rate of change of the perimeter is 4. Since x=1 is 2.1=2, the perimeter change rate is greater than that of the area. Field change. Since x=2 is 2.2=4, the perimeter change rate is equal to the rate of change of the area. Therefore, Mehmet's answer is not valid for these points." As can be seen, an argument based on considering the rate of change is used here for the cases x=1 and x=2, which contradicts the idea of the rate of change.

Some trainee teachers expressed the following opinions:

There are problems with reading the graph (incorrect reading).
I think Ali has problems understanding the chart.
Ali said that the rate of change in the area is different. However, the perimeter = 4x.
I'm afraid I have to disagree with Selma. Because Selma also provided equality in the rate of change of the area, but $A(x) = x^2$. $A'(x)=2x$, and the ratio is 10 from 4 to 6 and 14 from 6 to 8.
I have to disagree with Mehmet because going from 1 to 2, the rate of change of the environment is 2, while the rate of change in the area is 32. That is, the rate of change of the site is not always more significant than the rate of change of the perimeter.
Mehmet has correctly interpreted the graph. Since the slope of the area increase graph is greater than the slope of the perimeter increase graph, the area change is more significant than the perimeter change rate.
In the given question, two points are noted, and the accuracy of the graph for the rate of change of perimeter and area is found.
About the conceptual reasoning category, it was determined that the trainee teachers first discussed the methods of calculating the rate of change of the cartoon characters of contextualized concepts in the context of the previous question and also came to the information that the rate of change corresponds to the slope. As an example of these situations;

Ali created a table and arrived at the solution. Selma, on the other hand, concluded the formula. The answers are correct even though they try them in different ways. However, in the end, they both thought algebraically. They think differently in the graphical method because Ali only thinks about the points, while Selma thinks more generally.
The rate of change is the slope. Since the change in the environment is less than the change in the area, the slope of the graph will be less. We can also determine that the rate of change is the slope. The rate of change is the ratio of the change in the y's to the change in the x's. Therefore, rate of change equals equals$\mathrm{one} = \frac{y_1-y_2}{x_1-x_2}$. The graph shows that tan$\alpha = \frac{y_1-y_2}{x_1-x_2}$.

Mehmet correctly interpreted the graph and calculated the rate of change of area and perimeter for the points (6,36) and (4,16). To perform this calculation, he used the slope.

The opinions expressed show the students' use of their knowledge of the subject both to analyze the accuracy of the processes performed by the characters and to make predictions about alternative situations using the cartoon data. In addition, some trainee teachers made a joint judgment in questioning the answers given by the characters as a group. According to their joint decision, although Selma and Ali made the operationally correct types of changes, they stated that they had difficulty interpreting the graphs and were insufficient.

o We discussed Salman's response. As a group, we decided that he was wrong.
o Although Selma and Ali solved the question operationally, they were wrong when examining it graphically.
o We discussed Selma's answer. I think Selman is wrong.
o We think it is also because she is deficient in reading graphs.

From the opinions of the teachers-in-training, it can be said that the characters differentiate the plots and reveal their perspectives on the explanations made by each character, especially their ability to interpret graphs and solve questions in different ways. In addition, the explanation "Perimeter can be represented by a linear function, but a linear function cannot represent the area." It can be said that the teacher who does this has achieved conceptual knowledge because he differentiates the representation of the area from that of the perimeter concerning the linear function.

3. **Can they establish a relationship between the rate of change and the derivative in the case of graphical representation?**

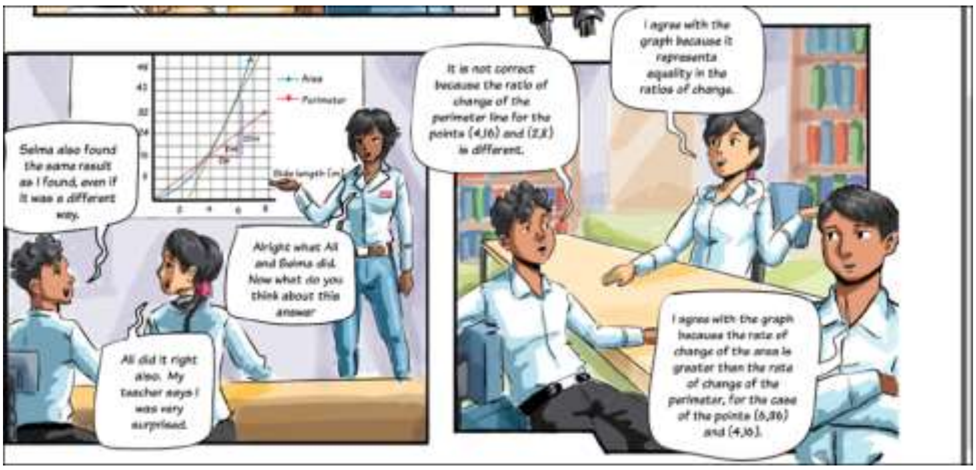

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Figure 2

Regarding the category on the interpretation of reasoning, the trainee teachers determined that the rate of change in the graph is equal to the slope with $\tan\alpha = \frac{y_1 - y_2}{x_1 - x_2}$ and stated that the graph will facilitate finding the rate of change at the given points, as well as that the results found by one of the characters without drawing the graph are in agreement with the graph. In this context, the students' opinions are as follows:

The graph quickly determines the perimeter and area change rate at the given points.

The results found without drawing the graph also support the graph.

In the graph, tan$= \frac{y_1 - y_2}{x_1 - x_2}$results.

Now, if we analyze the results with attention to the conceptual reasoning, most of the trainee teachers were able to explain with their reasons why Mehmet, who is one of the characters in the contextualized conceptual vignette, said the right thing and Selma said the wrong thing. As an example of this situation:

"I agree with Mehmet. Because for points (6,36) and (4,16), the rate of change of area will be greater than the rate of change of perimeter. If the area function is expressed as $A(x) = x^2$ and the perimeter function as Ç=4x, The rate of change of $A(x) = x^2$ will always be greater than Ç = 4x. When we look at its derivatives, we see Ç'(x)=4. So the rate of change in the environment will always be 4. The area will always be twice the area of the variable x, depending on x. Therefore, the rate of change in area is greater than the rate of change in the environment. "

"I disagree with Selma. Because Selma also provided the equality in the rate of change of Area, but $A(x) = x^2$. A'(x)=2x, and the ratio is ten from 4 to 6 and 14 from 6 to 8.

"I disagree with Ali because the rate of change in the environment is constant for all 4 m/m points. A linear equation represents it on the graph."

Additionally, it is noted that some teacher responses are evidence of both interpreting reasoning and conceptual questioning:

"The rate of change is the slope. Since the change in the environment is less than the change in the area, the slope of the graph will be less. We can also determine that the rate of change is the slope. The rate of change is the ratio between our change in y and x. So the rate of change $= \frac{y_1 - y_2}{x_1 - x_2}$ (conceptual meaning)."

"The slope of the tangent to the area graph is greater than the slope of the line graph of the perimeter. I agree with this view, as it gives the rate of change of the slopes. However, it is not a general acceptance; it is valid for the specified points."

"Because the rate of change is $y' = \frac{f(y)-f(y_0)}{x-x_0} = \frac{36-16}{6-4} = \frac{20}{2} = 10$ "

"When we calculate the rate of change of the environment in this way, it becomes $\frac{24-16}{6-4} = \frac{8}{2} = 4$. The truth of this thought is revealed."

"The rate of change of the area gives different results between different points, while the rate of change of the environment is constant."

In addition, the contextualized conceptual vignettes offered the opportunity to reveal alternative ideas in some teachers-in-training. These situations are described below:

(a) How to calculate the rate of change for a single point. For this situation, the teacher trainee said, "When we examine the graph, we find that the rate of change of the area is 2x and the rate of change of the surround is 4. Since 2.1=2 at x=1, the rate of change of the perimeter is greater than that of the area. Since 2.2=4 at x=2, the rate of change of perimeter and the rate of change of area are equal. Therefore, Mehmet's answer is invalid for these points." expression is an example.

b) When calculating the rate of change, the difference between the variables has not been considered when extracting the function values for the two points. As an example of this situation, "I disagree with Mehmet. Because going from 1 to 2, the rate of change of the environment is 2, while the rate of change of the area is 3/2. That is, the rate of change of the area is not always greater than the rate of change of the environment".

Although the rate of change is associated with the slope and a correct idea is put forward, it is not realized that this situation is valid for linear functions, and it is thought that the same idea is valid for all functions. Here it was also not noticed that the slope of the tangent line drawn at each point changed since the area curve is not linear but a parabolic curve. Here

"I agree with Selma. Because when we look at the slopes of the functions at those points on the graph, it appears that the rate of change of Area and Perimeter is equal to the slopes."

"I agree with Selma. Because the graph fully expresses the changes in the area and the surroundings. As for the rate of change, when the rate of change of Perimeter is equal to the slope of the line, but the rate of change of Area does not increase linearly, we can draw a tangent to find the rate of change."

It is calculated by the slope of the tangent of the area curve at that point on the graph ($\frac{20}{2} = 10$). The slope of the line graph of the perimeter ($\frac{8}{2} = 4$) gives the rate of change."

On the other hand, some teachers-in-training wondered whether the values the characters in the conceptualized conceptual vignette had previously found operational and the values in the graph coincided rather than focusing on the graphical interpretation of the rate of change of the perimeter and area functions given in the graph. In this context:

"I disagree with Mehmet. The fact that the rate of change of the area is greater than the rate of change of the surround is not enough to make the graph correct.

I agree with Selma; she wrote the professor's example based on one variable. She compared the results of the different variables on the graph drawn and saw that equality was achieved in the correct proportions.

I agree with Selma. Selma Because Selma's comments compared the formula and the graph.

Selma Because she is right. The area and perimeter graphs are drawn correctly, and the rate of change is correct."

It is observed that using contextual, conceptual vignettes provides an opportunity for students to become aware of mathematical concepts and thus generate ideas in line with mathematics. The students, who identified the operational differences between the characters of Selma and Ali in the contextual vignettes, did not hesitate to discuss their statements' accuracy, questioned the understanding of the graph, and associated the rate of change with the concept of the tangent. They intended to explain the effect of increasing area on cost, starting from the inflection point x=4, making it clear that the rate of change is related to the concept of the tangent with the graph. However, although the trainee teachers established the relationship between the concept of slope and rate of change, they had yet to learn of the effect of area change on cost.

On the other hand, a group of trainee teachers stated that the situation could not be understood with only one point in understanding the graph (for the concept of rate of change), and it is necessary to look at more than one point. It is believed that the trainee teachers who expressed this opinion structured the concept of rate of change not conceptually but intuitively because two points are necessary and sufficient conditions for the graph analysis.

Regarding the contextualized concept cartoon as a teaching tool, the teachers' opinions show that because of the training received so far, they have correctly structured the rate of change and associated it with the concept of derivative. Therefore, contextualized concept drawings play a crucial didactic role as course material. The following are the opinions of other teachers on this situation:

I agree with the graph because, at points (6,36) and (4,16), the rate of change in the area is more significant than in the environment. (joins Mehmet).

I agree with Mehmet's answer. Because as the length of the edge increases, the area's rate of change increases, which is correctly shown in the graph.

I agree with Mehmet because the rate of change of the area is greater than the rate of change of the perimeter in all cases except when the side length is equal to 1.

I agree with Mehmet because it is clear from examining the graph that the rate of change of the area is greater than that of the perimeter.

I agree with Mehmet because the rate of change in the area is greater than the rate of change in the surround.

I agree with Mehmet; I agree with Selma only on the rate of change in the environment.

I agree with Mehmet. Since the area is $x^{2,}$ the changes of the perimeter (4x) will be constant (4) while its change keeps increasing (2x). Therefore, the graph is correct.

I agree with Mehmet because at points (6,36) and (4,16), the rate of change of area is 10 $\frac{\mathrm{m}^2}{\mathrm{m}}$ the rate of change of perimeter is 4 $\frac{\mathrm{m}}{\mathrm{m}}$ of the perimeter.

I agree with Mehmet. Because when I look at the slopes of the graph, I see that the slope of the perimeter graph is 4, and the slope of the area graph is 10. So, since 10 > 4, Mehmet's statement is correct.

2x >4 provides the desired state (When I put the points in place).

Area → x^2 Derivative →2x

Area →4x Derivative →4

What Selma says is true for the environment.

I am afraid I have to disagree with that, Ali. Because the rate of change of the perimeter length in the graph is not different. Therefore, there is no error.

Ali said the rate of change in the environment is different. However, the perimeter = 4x.

I am afraid I have to disagree with Ali because the rate of change in the environment at those points is the same.

I thought the same as Selma and Ali. Only Selma has made it functional by using area and environment as varies.

According to the category of contextualized concept vignettes as a teaching tool, one group asked questions about explaining the importance of contextualized concept vignettes in their classroom context, as well as the value they placed on this material for their learning, of which the following examples are provided:

Why are contextualized concept cartoons critical?

Why do we need conceptual cartoons?

Contextualized conceptual cartoons allow us to convey a topic through cartoons.

Thanks to contextualized concept cartoons, I could more easily understand the essence of the rules with the help of different representations.

After the topic of the rate of variation, the issue of derivatives is explained. For the students to understand the definition of the slope of the derivative, the rate of change must be emphasized. If asked what is $\tan\alpha$ from the graphs, students can see this relationship more quickly. They can go from rate of change to instantaneous change and from slope to tangent slope.

To understand the accuracy of the graph, it is necessary to look at more than one point; it cannot be understood by looking at only one point.

Some views at the end of the vignette help to realize the misconceptions. It shows the difficulties posed by various mathematical notations.

"The dilemma disappears if you give information about a topic explained by falling into the contextual concept vignette."

Conclusion

The contextualized conceptual cartoon contributed to the generation of questions in the teachers in training, and they have a pedagogical value as an alternative to developing the theme, in this case, the reason for the change.
In the participants' opinion, this cartoon is a didactic strategy through which different ideas are proposed that can be transmitted more efficiently.
This type of didactic tool is an alternative expression in the classroom about derivatives or other related topics. The participants highlight this aspect as a help in strengthening the construction of knowledge and memory.

The teachers in training have learned that: instantaneous change is a derivative, the derivative is the slope, and they have related the tangent of α to the slope based on the students' graphs.

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